

Mathematics Tutorial 1S4

Coupled ODEs

A coupled system of ordinary differential equations (ODEs) involves a relation between two dependent variables x and y (representing the population levels of two species, for example) and their derivatives with respect to an independent variable t . First order derivatives often refer to individual specie growth rates with variable t indicating the time. A general first order system would relate

$$t, \quad x, \quad y, \quad \frac{dx}{dt}, \quad \frac{dy}{dt}.$$

Example 4 of Section 8.10 on page 546 of *Contemporary Linear Algebra*, John Wiley, by Howard Anton and Robert Busby [Hamilton Library 512.5 P3] provides a solution overleaf of the differential equations appearing in the first problem, albeit with a different notation where x and y are denoted by y_1 and y_2 , respectively. You are asked to verify the solution.

An ‘initial value problem’ refers to an ODE system where the values of x and y , the specie populations for example, are given at a particular value of time t . The constants appearing in the solution of the system may be determined from such initial values of x and y .

Answer the following in relation to a system of coupled linear first order ordinary differential equations. (Anton and Rorres, 8e, page 422, section 9.1 give one way of solving Q1. The method of Anton and Busby may be simpler).

1. Confirm by substitution that, for any values of the constants a and b , the expressions

$$x = a e^{2t} - b e^{-3t}$$

$$y = a e^{2t} + 4b e^{-3t}$$

solve the following system of coupled linear differential equations

$$\frac{dx}{dt} = x + y$$

$$\frac{dy}{dt} = 4x - 2y.$$

2. Find the solution of the system of coupled linear differential equations

$$\frac{dx}{dt} = -4x + y$$

$$\frac{dy}{dt} = 4x - 4y$$

given the initial values $x(0) = 7$ and $y(0) = 10$. Would these equations describe a symbiotic system? Do either of the species survive extinction? Explain.