The space of neurons
James B. Gillespie, Conor J. Houghton, Trinity College Dublin
gillesjb@tcd.ie and houghton@maths.tcd.ie

Abstract
A metric can be used to measure the distance between spike trains corresponding to the same stimulus. The distribution of these noise distances can be plotted, thus associating a noise distribution with each neuron. This embeds each neuron in a distribution space called a statistical model. Low dimensional embedding methods such as isomap can be applied to the set of noise distributions to estimate both the dimension and structure of this neuronal space.

The method is useful as a technique for investigating models of neuronal noise. It allows neuronal noise to be visualized both within a network of neurons and throughout networks of neurons. Results also infer the existence of two different neuron types within the datasets examined.

Overview
A metric can be used to define a distance between spike trains. This turns the space of spike trains into a metric space [1]. If, for a particular neuron, only distances between spike trains corresponding to the same stimulus are calculated, then the resulting noise distribution of distances reflects the noise associated with that particular neuron.

The L2 distances between the estimated noise distributions of the different neurons can be used to construct a matrix of inter-neuronal distances. The low dimensional embedding method isomap [2] can then be applied to estimate both the dimension and structure of the corresponding neuronal space.

The space of spike trains
The geometry of the space of spike trains is derived from a distance function defined on the spike trains. In the van Rossum metric [3] the spike train, considered as a list of spike times:

\[ u = \{ u_1, u_2, \ldots, u_n \} \]

is mapped to a real function,

\[ u \mapsto f(t; u) = \sum_{i=1}^{n} h(t - u_i) \]

using a kernel:

\[ h(t) = \begin{cases} 0 & t < 0 \\ e^{-t^2/T} & t \geq 0 \end{cases} \]

The distance between two spike trains is taken to be the distance between the two corresponding functions, using the standard L2 metric on the space of real functions.

\[ d(u_1, u_2) = \sqrt{\int [f(t; u_1) - f(t; u_2)]^2 dt} \]

where \( T \) is a time-scale which parameterizes the metric.

Data
Results here are obtained for extracellular spike trains, obtained from eight regions L, L1, L2a, L2b, L3, M1d, CM and OV of zebra finch auditory forebrain during playback of conspecific song.

Twenty responses from a particular neuron were recorded for each song, equivalent to 190 associated noise distances.

Noise distribution and statistical model
If, for a particular neuron, only distances between spike trains corresponding to the same stimulus are calculated, the resulting distances reflect the level of noise associated with that particular neuron. The distribution of these noise distances can be plotted using Kernel Density Estimation, thus associating a noise distribution with each neuron.

\[ P : \chi \rightarrow \mathbb{R} \]

\[ S = \{ p_\xi = p(x; \xi) \xi = [\xi^1, \ldots, \xi^n] \in \Xi \} \]

This is analogous to a manifold, with free parameters representing a coordinate system. Thus by associating a noise distribution with each neuron we are embedding the neurons in this space.

Isomap
The hypothesis that neurons are embedded in some low-dimensional space can be tested and visualized using isomap.

Input: the set of Euclidean distances between all noise distributions associated with a particular set of neurons.

Output: a plot of the geodesic distances between points, where each point represents a neuron, and the estimated dimensionality of the space.

The axis of the resulting graph should identify the free parameters of the noise distributions.

Results: The space of neurons
Isomap was applied to 450 noise distributions corresponding to neurons from eight different regions of the auditory forebrain of zebra finch. The dimensionality of the space was found to be approximately two dimensional.

The visualization of the space of neurons can be extended further by plotting a cell colour corresponding to \( \mu \) the average noise distance from a particular cell’s noise distribution.

Results: Properties of neuronal space
It is suggested that the bunching of high noise neurons seen in the space represents the identification of a second cell type. This distinction can be seen clearly if a density plot of \( \mu \) for a particular region in the auditory forebrain is generated.

References

Acknowledgements
JBG wishes to thanks the Irish Research Council of Science, Engineering and Technology for an Embark Postgraduate Research Scholarship. CJH wishes to thank the James S McDonnell Foundation for their Scholar award. The authors would like to thank the Theunissen lab for allowing the use of their electrophysiological data, provided on the C.R.C.N.S. datasharing site.

Output
Input
Isomap
Kernel Density Estimation
Noise distribution
Statistical model
Van Rossum metric
Central and Peripheral Auditory Systems
Neural Computation
The space of neurons
Isomap was applied to 450 noise distributions corresponding to neurons from eight different regions of the auditory forebrain of zebra finch. The dimensionality of the space was found to be approximately two dimensional.

The visualization of the space of neurons can be extended further by plotting a cell colour corresponding to \( \mu \) the average noise distance from a particular cell’s noise distribution.

Results: Properties of neuronal space
It is suggested that the bunching of high noise neurons seen in the space represents the identification of a second cell type. This distinction can be seen clearly if a density plot of \( \mu \) for a particular region in the auditory forebrain is generated.

References

Acknowledgements
JBG wishes to thanks the Irish Research Council of Science, Engineering and Technology for an Embark Postgraduate Research Scholarship. CJH wishes to thank the James S McDonnell Foundation for their Scholar award. The authors would like to thank the Theunissen lab for allowing the use of their electrophysiological data, provided on the C.R.C.N.S. datasharing site.