1M01 Mathematical Methods 2010–11
Calculus tutorial exercise sheet 8

1. Simplify and then differentiate the following with respect to x: Notes: 66–69

   (a) $e^{2\ln(x)}$ (b) $\sqrt{e^{1+2x^3}}$ (c) $\ln\left(\dfrac{\sqrt{x}}{\cos(x)}\right)$

   Hint: these are easier to do if you simplify before differentiating! Remember that $\dfrac{d}{dx}(e^x) = e^x$, and $\dfrac{d}{dx}\big(\ln(x)\big) = \dfrac{1}{x}$.

2. Compute: Notes: 66,67

   (a) $\displaystyle\int e^{-x/2} + 1\, dx$ (b) $\displaystyle\int_0^{\ln(2)} \frac{2}{e^x}\, dx$ (c) $\displaystyle\int te^{3t}\, dt$ Notes: 60–63

   Hint: for (c), try integration by parts with $u = t$ and $dv = e^{3t}\, dt$.

3. The size of a population of bacteria is modelled by $P(t) = 10^8 \times e^{0.02t}$ where t is time, in hours. Notes: 72,73

   (a) Find the initial population and the generation time.

   (b) When does the population reach 10^9 bacteria?

4. The radioactive element Carbon-14 has a half-life of 5750 years. The amount of Carbon-14 in a fossil decays according to the differential equation Notes: 74,75

$$\frac{dN}{dt} = -kN,$$

   where $N = N(t)$ is the amount of Carbon-14 present in the fossil at time t, measured in years since the fossilised organism died, and k is a positive constant.

   Find the value of the constant k, and compute the age of a fossil which has lost 95% of its Carbon-14.