1 Maths methods Calculus ex Sheet 7

(a) $\frac{d}{dx}\left(\frac{x}{1-x}\right)$

$u = x \quad v = 1-x$

$\frac{du}{dx} = 1 \quad \frac{dv}{dx} = -1$

$$= \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2} = \frac{(1-x).1 - x.(-1)}{(1-x)^2}$$

$$= \frac{1-x+x}{(1-x)^2} = \frac{1}{(1-x)^2}$$

(b) $\frac{d}{dx}\left(3x - x^8\cos(x)\right)$

$$= \frac{d}{dx}(3x) - \frac{d}{dx}(\underbrace{x^8}_{u}\underbrace{\cos(x)}_{v})$$

$u = x^8 \quad v = \cos(x)$

$\frac{du}{dx} = 8x^7 \quad \frac{dv}{dx} = -\sin(x)$

$$= 3 - \frac{d}{dx}(uv)$$

$$= 3 - \left(u\frac{dv}{dx} + v\frac{du}{dx}\right) = 3 - \left(x^8.(-\sin(x)) + \cos(x).8x^7\right)$$

$$= 3 + x^8\sin(x) - 8x^7\cos(x)$$

(c) $\frac{d}{dx}\left(\frac{\sin(2\pi x)}{5} + 1\right) = \frac{1}{5}\frac{d}{dx}\left(\sin(\underbrace{2\pi x}_{u})\right) + 0$

$u = 2\pi x$

$\frac{du}{dx} = 2\pi$

$$= \frac{1}{5}\frac{d}{dx}(\sin(u))$$

$$= \frac{1}{5}\frac{d}{du}(\sin(u)) . \frac{du}{dx}$$

$$= \frac{1}{5}\cos(u) . 2\pi = \frac{2\pi}{5}\cos(2\pi x)$$

$u = \cos(4x)$ | $v = \sin(x)$

$\frac{du}{dx} = -\sin(4x).4 = -4\sin(4x)$ | $\frac{dv}{dx} = \cos(x)$

(d) $\frac{d}{dx}\left(-2\cos(4x)\sin(x)\right)$

$= -2\frac{d}{dx}\left(\underbrace{\cos(4x)}_{u}\ \underbrace{\sin(x)}_{v}\right)$

$= -2\ \frac{d}{dx}(uv) = -2\left(u\frac{dv}{dx} + v\frac{du}{dx}\right)$

$= -2\left(\cos(4x)\cdot\cos(x) + \sin(x)\cdot\left(-4\sin(4x)\right)\right)$

$= \underline{8\sin(x)\sin(4x) - 2\cos(4x)\cos(x)}.$

$u = x^3 - 2$

$\frac{du}{dx} = 3x^2$

(e) $\frac{d}{dx}\left(\cos(\underbrace{x^3-2}_{u})\right) = \frac{d}{dx}\left(\cos(u)\right)$

$= \frac{d}{du}\left(\cos(u)\right)\cdot\frac{du}{dx} = -\sin(u)\cdot 3x^2 = \underline{-3x^2\sin(3x^2-2)}$

$u = x^3$ | $v = e^{2x}$

$\frac{du}{dx} = 3x^2$ | $\frac{dv}{dx} = 2e^{2x}$

(f) $\frac{d}{dx}\left(\underbrace{x^3}_{u}\underbrace{e^{2x}}_{v}\right) = \frac{d}{dx}(uv)$

$= u\frac{dv}{dx} + v\frac{du}{dx}$

$= x^3.2e^{2x} + e^{2x}.3x^2$

$= \underline{x^2e^{2x}(2x+3)}.$

②

(a) $\int 3x^5\sqrt{x^6-2}\,dx$

$u = x^6-2$
$du = 6x^5\,dx$
$\therefore$ ~~2du = 3~~
$\frac{1}{2}du = 3x^5\,dx$

$= \frac{1}{2}\int \sqrt{u}\,du$

$= \frac{1}{2}\int u^{1/2}\,du = \frac{1}{2}\frac{u^{3/2}}{3/2} + C$

$= \frac{1}{2}\cdot\frac{2}{3}u^{3/2} + C = \frac{1}{3}u^{3/2} + C$

$= \frac{1}{3}(x^6-2)^{3/2} + C$

(b) $\int 1500t(5t^2+4)^{99}\,dt$

$u = 5t^2+4$
$du = 10t\,dt$
$150du = 1500t\,dt$

$= 150\int u^{99}\,dt$

$= 150\frac{u^{100}}{100} + C = \frac{3}{2}u^{100} + C = \frac{3}{2}(5t^2+4)^{100} + C$

(c) $\int \underbrace{1500t}_{u}\,\underbrace{(5t+4)^{99}\,dt}_{dv}$

$u = 1500t$
$du = 1500\,dt$

~~dv = ...~~
$dv = (5t+4)^{99}$
$v = \frac{(5t+4)^{100}}{500}$

$= \int u\,dv = uv - \int v\,du$

$= 1500t\cdot\frac{(5t+4)^{100}}{500} - \int\frac{(5t+4)^{100}}{500}\cdot 1500\,dt$

$= 3t(5t+4)^{100} - 3\int(5t+4)^{100}\,dt$

$= 3t(5t+4)^{100} - \frac{3}{101\times 5}(5t+4)^{101} + C$

$= 3t(5t+4)^{100} - \frac{3}{505}(5t+4)^{101} + C$

(d) $$\int \underbrace{18x}_{u}\ \underbrace{\cos(3x+4)\,dx}_{dv}$$

$u = 18x$ $\quad$ $dv = \cos(3x+4)$

$du = 18\,dx$ $\quad$ $v = \frac{1}{3}\sin(3x+4)$

$$= \int u\,dv = uv - \int v\,du$$

$$= 18x \cdot \frac{1}{3}\sin(3x+4) - \int \frac{1}{3}\sin(3x+4) \cdot 18\,dx$$

$$= 6x\sin(3x+4) - 6\int \sin(3x+4)\,dx$$

$$= 6x\sin(3x+4) - 6 \cdot \frac{1}{3}\left(-\cos(3x+4)\right) + C$$

$$= 6x\sin(3x+4) + 2\cos(3x+4) + C$$

(e) $$\int \frac{x^2(2x^3+5)^{12} - 4}{7}\,dx$$

$$= \frac{1}{7}\int x^2(2x^3+5)^{12} - 4\,dx$$

$$= \frac{1}{7}\int x^2\underbrace{(2x^3+5)}_{u}{}^{12}\,dx - \frac{4}{7}x$$

$u = 2x^3+5$

$du = 6x^2\,dx$

$\frac{1}{6}du = x^2\,dx$

$$= \frac{1}{7}\cdot\frac{1}{6}\int u^{12}\,du - \frac{4}{7}x$$

$$= \frac{1}{42}\,\frac{u^{13}}{13} - \frac{4}{7}x + C$$

$$= \frac{1}{546}(2x^3+5)^{13} - \frac{4}{7}x + C$$