Solutions to Calculus tutorial ex. sheet 6.

(1)

(a) $360° = \frac{360°}{180°} \times \pi = \underline{2\pi \text{ radians}}$

(b) $35° = \frac{35°}{180°} \times \pi = \underline{\frac{7}{36}\pi \text{ radians}}$

(c) $-1.5\pi = \frac{-1.5\pi}{\pi} \times 180° = -1.5 \times 180° = \underline{-270°}$

(d) $2.2 \text{ radians} = \frac{2.2}{\pi} \times 180° = \left(\frac{396}{\pi}\right)° \approx \underline{\underline{126.051°}}$

(2)

(a) Day length is the same at the same time of year;

so $L(t+1) = L(t)$, so $L(t)$ is periodic with period 1 year.

1 year later than time t, so same time of year

(b) Period: $p = 1$ so $b = \frac{2\pi}{p} = 2\pi$

Average value: $k = 12$

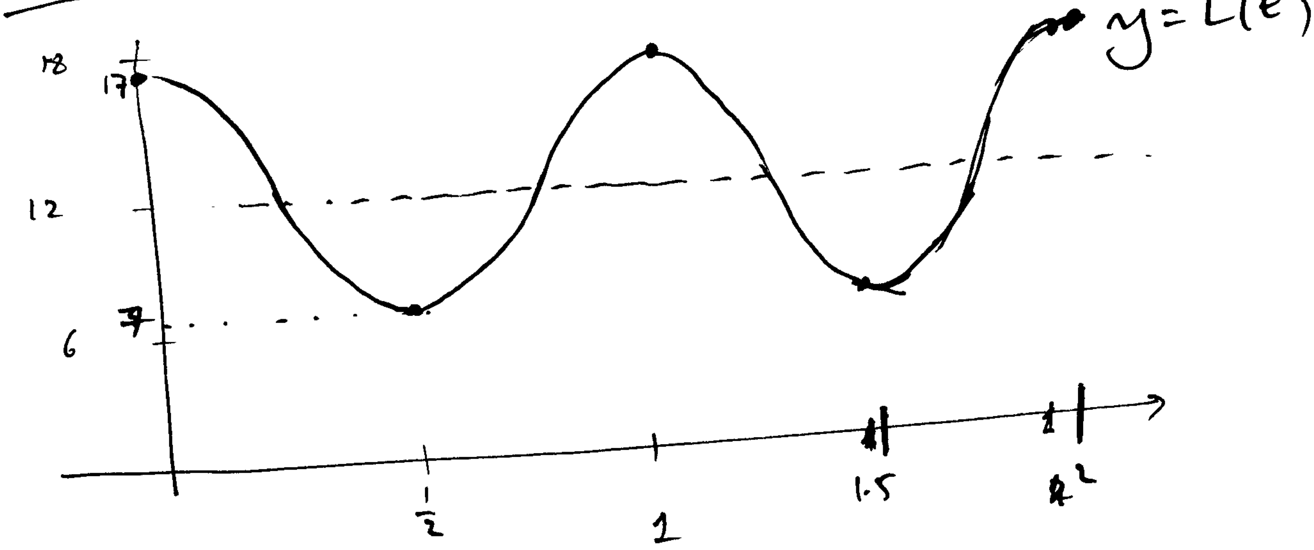
Amplitude: max value − average value

$a = 17 - 12 = 5$.

So $L(t) = k + a\cos(bt)$ cos (& not sin) since when $t=0$, have longest day so $L(t)$ is @ max value

So $\underline{L(t) = 12 + 5\cos(2\pi t)}$.

<u>Sketch:</u>



(d) When $t = \frac{2}{12}$, $L(t) = 12 + 5\cos\left(2\pi \times \frac{2}{12}\right)$

$= 12 + 5\cos\left(\frac{\pi}{3}\right) = \underline{\underline{14.5 \text{ hours}}}$

③ (a) $\frac{d}{dt}\left(-4\cos(3t^2)\right) = -4(-\sin(3t^2)).6t$

$\boxed{\begin{array}{l} u = 3t^2 \\ \frac{du}{dt} = 6t \end{array}}$

$= \underline{24t\sin(3t^2)}$.

(b) $y = x^2 + \sin(2x + \pi)$

$u = 2x + \pi$
$\frac{du}{dx} = 2$

$\frac{dy}{dx} = 2x + \cos(2x+\pi).2$

So $\left.\frac{dy}{dx}\right|_{x=1} = 2 \times 1 + \cos(2+\pi) \times 2 \approx 2.83229$

(c) $\frac{d}{dx}\left(\cos\left(\underbrace{x^{-1} - \sin(x)}_{u}\right)\right)$

$u = x^{-1} - \sin(x)$

$\frac{du}{dx} = -x^{-2} - \cos(x)$

$= -\left(x^{-2} + \cos(x)\right)$

$= \frac{d}{dx}\left(\cos(u)\right)$

$= -\sin(u) \cdot \frac{du}{dx}$

$= -\sin\left(x^{-1} - \sin(x)\right) \cdot \left(-\left(x^{-2} + \cos(x)\right)\right)$

$= \underline{\left(x^{-2} + \cos(x)\right) \cdot \sin\left(x^{-1} - \sin(x)\right)}.$

(4)

(a) $\int \frac{5\sin(x)}{3}\, dx = \int \frac{5}{3} \cdot \sin(x)\, dx$

$= \frac{5}{3} \int \sin(x)\, dx = \underline{-\frac{5}{3}\cos(x) + C}$

(b) $y_{av} = \frac{1}{\pi/2 - 0} \int_0^{\pi/2} \cos(x)\, dx$

$= \frac{1}{\frac{\pi}{2}} \left[\sin(x)\right]_0^{\pi/2} = \frac{2}{\pi}\left(\sin\left(\frac{\pi}{2}\right) - \sin(0)\right)$

$= \frac{2}{\pi}(1 - 0) = \underline{\underline{\frac{2}{\pi}}}$

(c) $\int_0^{\pi/4} 3 - 2\sin(2t)\, dt$

$u = 2t$

$du = 2dt$

t	$u=2t$
0	0
$\pi/4$	$\pi/2$

$= \int_0^{\pi/4} 3\, dt - \int_0^{\pi/4} 2\sin(2t)\, dt$

$= \left[3t\right]_0^{\pi/4} - \int_0^{\pi/2} \sin(u)\, du$

$= 3 \times \frac{\pi}{4} - 0 - \left[-\cos(u)\right]_0^{\pi/2} = \frac{3\pi}{4} + \left(\cos\left(\frac{\pi}{2}\right) - \cos(0)\right) = \underline{\underline{\frac{3\pi}{4} - 1}}$