1M01 Calculus tutorial sheet 5 Solutions.

(1)

(a)

$$\sqrt{\frac{8x^2+8y^2}{2}} = \sqrt{4x^2+4y^2}$$ (cancelling 2's on top & bottom)

$$= \sqrt{4(x^2+y^2)}$$ (factorising)

$$= \left(4(x^2+y^2)\right)^{1/2}$$

$$= 4^{1/2}\left(x^2+y^2\right)^{1/2}$$

$$= 2\left(x^2+y^2\right)^{1/2}$$

(b)

$$\left(\sqrt[4]{81x^3y^6}\right)^3$$

$$= \left(\left(81x^3y^6\right)^{1/4}\right)^3$$

$$= \left(81^{1/4}\left(x^3\right)^{1/4}\left(y^6\right)^{1/4}\right)^3$$

$$= \left(3\ x^{3/4}\ y^{6/4}\right)^3$$

$$= 3^3\ x^{9/4}\ y^{18/4} = 27\,x^{9/4}\,y^{9/2}.$$

② (a) $\frac{d}{dx}(\sqrt{x}) = \frac{d}{dx}(x^{1/2}) = \underline{\frac{1}{2}x^{-1/2}}$

$\frac{d}{dx}\left(\frac{1}{\sqrt{x}}\right) = \frac{d}{dx}\left(x^{-1/2}\right) = \underline{-\frac{1}{2}x^{-3/2}}$

(b) $\frac{d}{dx}\left(4\sqrt{x^3} - \frac{1}{x}\right) = \frac{d}{dx}\left(4x^{3/2} - x^{-1}\right)$

$= 4 \cdot \frac{3}{2}x^{1/2} - (-1)x^{-2}$

$= \underline{6x^{1/2} + x^{-2}}$

(c) $\frac{d}{dt}\left((t^3+4)^{2010}\right) = 2010(t^3+4)^{2009} \cdot 3t^2$

$= \underline{6030t^2(t^3+4)^{2009}}$

(d) $\left.\frac{dy}{dx}\right|_{x=1} = \frac{1}{3}(2x^9-x)^{-2/3} \cdot (18x^8-1)\Big|_{x=1} = \frac{1}{3}(2-1)^{-2/3} \cdot (18-1)$

$= \underline{\underline{\frac{17}{3}}}$

③ (a) $\int \frac{5t^{1/5}}{2} - \frac{10}{t^3}\,dt = \int \frac{5}{2}t^{1/5} - 10 \cdot t^{-3}\,dt$

$= \frac{5}{2} \cdot \frac{5}{6}t^{6/5} - 10 \cdot \frac{1}{(-2)}t^{-2} + C$

$= \underline{\frac{25}{12}t^{6/5} + 5t^{-2} + C}$

(b) $\int_1^2 3x^{1.6} + 2\,dx = \left[\frac{3x^{2.6}}{2.6} + 2x\right]_1^2$

$= \left(\frac{3 \cdot 2^{2.6}}{2.6} + 2\times2\right) - \left(\frac{3 \cdot 1^{2.6}}{2.6} + 2\times1\right)$

$= 10.996 - 3.1539 = \underline{\underline{7.842}}$ (to 3 dp)

3 (c). $f'(x) = x\sqrt{x^2+1}$, so

$$f(x) = \int f'(x)\,dx = \int x\sqrt{x^2+1}\,dx$$

$$= \int x(x^2+1)^{1/2}\,dx$$

$u = x^2+1$

$\frac{du}{dx} = 2x$

$x = \frac{1}{2}\frac{du}{dx}$

$$= \int \frac{1}{2}u^{1/2} \cdot \frac{du}{dx}\,dx$$

$$= \frac{1}{2} \cdot \frac{u^{3/2}}{3/2} + C = \frac{1}{3}u^{3/2} + C$$

$$= \frac{1}{3}(x^2+1)^{3/2} + C$$

Now $f(0) = 1$, so $\frac{1}{3}(0^2+1)^{3/2} + C = 1$, so $\frac{1}{3} + C = 1$, so $C = \frac{2}{3}$.

So $f(x) = \frac{1}{3}(x^2+1)^{3/2} + \frac{2}{3}$.

(d) $$\int \frac{-2x^2}{(4x^3-1)^2}\,dx$$

Let $u = 4x^3 - 1$

$\frac{du}{dx} = 12x^2$

so $-\frac{1}{6}\frac{du}{dx} = -2x^2$

$$= \int \frac{-\frac{1}{6}\frac{du}{dx}}{u^2}\,dx$$

$$= -\frac{1}{6}\int u^{-2}\frac{du}{dx}\,dx$$

$$= -\frac{1}{6} \cdot \frac{u^{-1}}{(-1)} + C$$

$$= +\frac{1}{6} \cdot \frac{1}{u} + C = \frac{1}{6(4x^3-1)} + C$$

④ $r = k_1 \sqrt[12]{w} = k_1 w^{1/12}$ (For some constant $k_1 > 0$)

& $n = k_2 w^{5/8}$ (—— " —— $k_2 > 0$)

so $w = \left(\frac{1}{k_1} r\right)^{12} = k_3 r^{12}$ (—— " —— $k_3 > 0$)

so $n = k_2 \cdot (k_3 r^{12})^{5/8}$

$= k_4 \, r^{12 \times 5/8}$ (—— " —— $k_4 > 0$)

$= \underline{k_4 \cdot r^{15/2}}$.

So n is directly proportional to $r^{15/2}$
(& $s = \frac{15}{2}$).