1M01 Mathematical Methods 2010–11

Calculus tutorial exercise sheet 5

1. Simplify the following expressions, writing your answers using powers rather than roots. Notes: 43–44

   (a) $\sqrt{\dfrac{8x^2+8y^2}{2}}$ (b) $\left(\sqrt[4]{81x^3y^6}\right)^3$

2. (a) Find the derivatives of $\sqrt{x}$, and $\dfrac{1}{\sqrt{x}}$. Notes: 45–46

   (b) Find $\dfrac{d}{dx}\left(4\sqrt{x^3}-\dfrac{1}{x}\right)$.

   (c) Compute $\dfrac{d}{dt}\left((t^3+4)^{2010}\right)$.

   (d) What is the slope of the tangent line to $y=(2x^9-x)^{1/3}$ at $x=1$?

3. (a) Compute $\displaystyle\int \frac{5t^{1/5}}{2}-\frac{10}{t^3}\,dt$. Notes: 47–48

   (b) Find $\displaystyle\int_1^2 3x^{1.6}+2\,dx$.

   (c) Find a function $f(x)$ so that $f'(x)=x\sqrt{x^2+1}$ and $f(0)=1$.

   (d) Compute $\displaystyle\int \frac{-2x^2}{(4x^3-1)^2}\,dx$.

4. A model for the cardiovascular system of mammals makes the following predictions about capillary blood vessels. Here, w is the mammal's weight, r is the typical radius of a capillary blood vessel and n is the total number of capillary blood vessels. Notes: 44

   - r is directly proportional to $\sqrt[12]{w}$
   - n is directly proportional to $w^{5/8}$

   Explain why n is directly proportional to r^s for some power s, and find s.