1MO1 Calculus tutorial 4 solutions

① $f(x) = x^3 + 4x - 6$

(a) $x_1 = -1$ so $y_1 = f(x_1) = (-1)^3 + 4(-1) - 6 = -1 - 4 - 6 = -11$

$x_2 = 2$ so $y_2 = f(x_2) = 2^3 + 4 \cdot 2 - 6 = 8 + 8 - 6 = 10$

So average rate of change is $\frac{y_2 - y_1}{x_2 - x_1} = \frac{10 - (-11)}{2 - (-1)} = \frac{21}{3} = \underline{\underline{7}}$

(b) $f'(3) = \frac{d}{dx}\left(x^3 + 4x - 6\right)\Big|_{x=3}$

$= \left(3x^2 + 4\right)\Big|_{x=3} = 3 \cdot 3^2 + 4 = \underline{\underline{31}}$

(c) slope of tangent @ $x = 0$ is $f'(0)$

$f'(0) = \frac{d}{dx}\left(x^3 + 4x - 6\right)\Big|_{x=0} = \left(3x^2 + 4\right)\Big|_{x=0} = \underline{\underline{4}}$

(d) value @ $x = 0$ is $f(0) = \left(x^3 + 4x - 6\right)\Big|_{x=0} = \underline{\underline{-6}}$

(e) av. value as $x: 0 \rightsquigarrow 3$ is

$$y_{av} = \frac{1}{3-0}\int_0^3 x^3 + 4x - 6 \, dx$$

$$= \frac{1}{3}\left[\frac{x^4}{4} + 2x^2 - 6x\right]_0^3$$

$$= \frac{1}{3}\left(\left(\frac{3^4}{4} + \underbrace{2 \cdot 3^2 - 6 \cdot 3}_{0}\right) - \underbrace{\left(\frac{0^4}{4} + 2 \cdot 0^2 - 6 \cdot 0\right)}_{0}\right)$$

$$= \underline{\underline{\frac{27}{4}}}$$

(2) $P(t) = \frac{1}{3}t^2 + 400$.

(a) initial pop is $P(0) = \underbrace{\frac{1}{3}\cdot 0^2}_{0} + 400 = \underline{\underline{400}}$

(b) after 12 months, pop is

$P(12) = \frac{1}{3}\cdot 12^2 + 400 = \underline{\underline{448}}$

(c) pop growth rate is $P'(t) = \frac{d}{dt}\left(\frac{1}{3}t^2 + 400\right)$

$= \underline{\underline{\frac{2}{3}t}}$

So after 12 months: $P'(12) = \frac{2}{3}\cdot 12 = \underline{\underline{8 \text{ aardvarks/month}}}$

(d) av. pop is av value of $P(t)$:

$$\frac{1}{12-0}\int_0^{12} P(t)\,dt = \frac{1}{12}\int_0^{12} \frac{1}{3}t^2 + 400\,dt$$

$$= \frac{1}{12}\left[\frac{1}{9}t^3 + 400t\right]_0^{12}$$

$$= \frac{1}{12}\left(\left(\frac{1}{9}\times 12^3 + 400\times 12\right) - 0\right)$$

$$= \frac{1}{9}\times 12^2 + 400 = \underline{416 \text{ aardvarks}}$$

(3)

$$\left.\begin{aligned} \text{Birth rate} &= 80 \\ \text{Death rate} &= 60 + 10t \end{aligned}\right\}$$

So pop growth rate is $P'(t) = 80 - (60 + 10t)$

ie. $\boxed{P'(t) = 20 - 10t}$.

So $P(t) = \int P'(t)\,dt$

$= \int 20 - 10t \; dt$

$= 20t - 5t^2 + C$

and initial value is 500, so

$P(0) = 500$, so $C = 500$.

Hence $\underline{P(t) = 20t - 5t^2 + 500}$

So after five years, population is

$P(5) = 20 \times 5 - 5 \times 5^2 + 500$

$= 100 - 125 + 500$

$= \underline{\underline{475}}$ antelopes