1M01 Mathematical Methods 2010–11
Calculus tutorial exercise sheet 4

1. Let $f(x) = x^3 + 4x - 6$. Find: Notes: 22,35,37,40

   (a) the average rate of change of $f(x)$ as x varies from -1 to 2.

   (b) the rate of change of $f(x)$ at $x = 3$.

   (c) the slope of the tangent line to $y = f(x)$ at $x = 0$.

   (d) the value of $f(x)$ at $x = 0$.

   (e) the average value of $f(x)$ as x varies from 0 to 3.

2. A population of aardvarks on an island is studied. The population size Notes: 38, 40 is found to be

   $$P(t) = \tfrac{1}{3}t^2 + 400$$

   where t is the time, in months, since the population was first studied.

   (a) When it was first studied, what was the aardvark population?

   (b) What was the aardvark population on the island twelve months after the study began?

   (c) What was the population growth rate 12 months after the study began?

   (d) What was the average population over the first 12 months of the study?

3. A herd of antelopes with an initial size of 500 falls victim to poachers. Notes: 38 The birth rate of the population holds steady at 80 antelopes per year, but the death rate of the population is given by

   $$D(t) = 60 + 10t$$

   where t is the time in years since the poaching began. Given that the population growth rate is

   $$P'(t) = \text{birth rate} - \text{death rate},$$

   find the size of the herd after five years of poaching.