1M01 Calculus tutorial sheet 3 solutions

① (a) $\frac{d}{dx}\left(5x^4-2x+3\right) = 5\cdot 4x^3 - 2\cdot 1 + 0$

$= \underline{\underline{20x^3-2}}$.

(b) $P(t) = 400t^4 + 3000$, so

$P'(t) = \frac{d}{dt}\left(400t^4+300\right) = 400\cdot 4t^3 + 0$

$= \underline{1600t^3}$

So $P'(2) = 1600t^3\big|_{t=2} = 1600\times 2^3$

$= 1600\times 8 = \underline{12800}$.

(c) $y = x(x-1)^2 = x(x-1)(x-1)$

$= x\left(x^2-x-x+1\right) = x\left(x^2-2x+1\right)$

$= x^3-2x^2+x$.

So $\frac{dy}{dx} = \frac{d}{dx}\left(x^3-2x^2+x\right) = 3x^2+4x+1$.

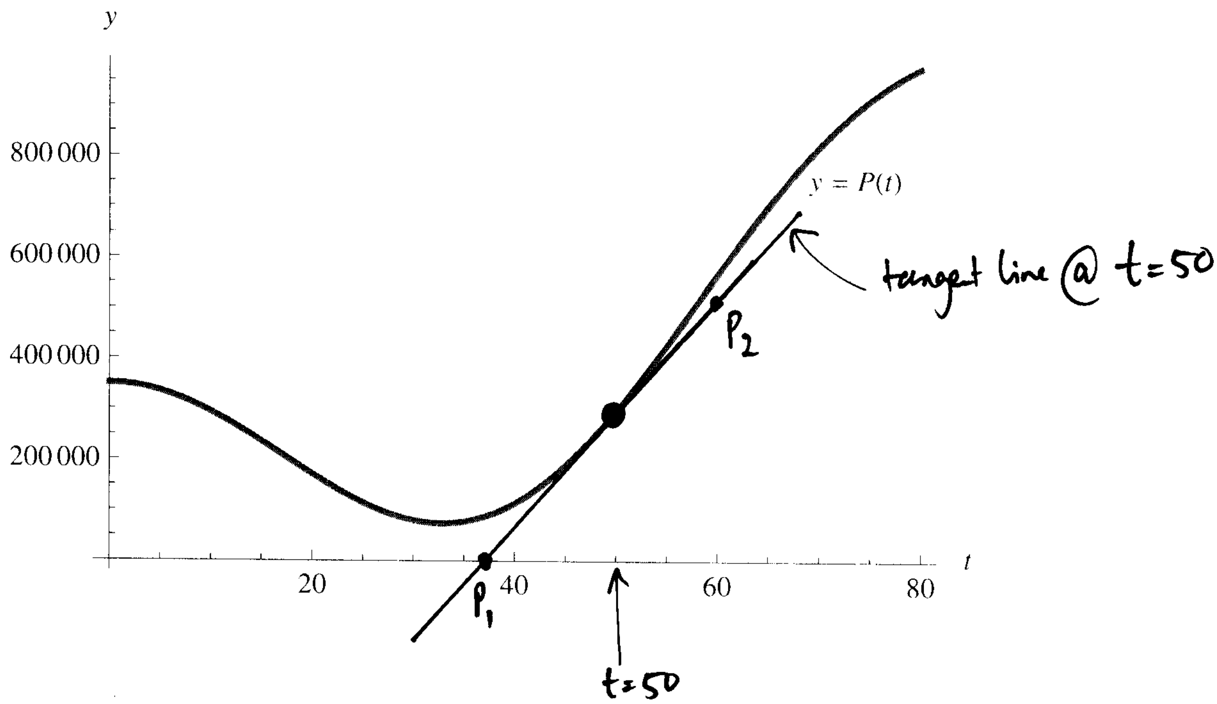
So $\frac{dy}{dx}\Big|_{x=1} = 3\cdot 1^2+4\cdot 1+1 = \underline{\underline{8}}$

(d) $y = x^2+5x-1$, so $\frac{dy}{dx} = 2x+5$, so

slope of tangent line to $y = x^2+5x-1$ @ $x=4$

is $\frac{dy}{dx}\Big|_{x=4} = 2x+5\Big|_{x=4} = 2\cdot 4+5 = \underline{\underline{13}}$.

(2)



Coords of P_1: $(37, 0) = (t_1, y_1)$ (approximately)
Coords of P_2: $(60, 550000) = (t_2, y_2)$

So growth rate @ $t = 50$

= slope of the tangent line

$$= \frac{y_2 - y_1}{t_2 - t_1} \approx \frac{550000 - 0}{60 - 37} = \frac{5.5 \times 10^5}{23}$$

$= 2.4 \times 10^4$ bacteria/hour (to 1 d.p.)

[You may come up with a slightly different answer, depending exactly how you draw your tangent line].

③ (a) $\int \frac{1}{2}x^3 - 4\,dx = \frac{1}{2}\int x^3\,dx - \int 4\,dx$

$= \frac{1}{2}\cdot\frac{x^4}{4} - 4x + C$

$= \underline{\frac{1}{8}x^4 \cancel{+} - 4x + C}$.

(b). $\int 7(t-4)^2\,dt = 7\int (t-4)^2\,dt$

$= 7\int t^2 - 8t + 16\,dt$ (expanding the brackets)

$= 7\left(\frac{1}{3}t^3 - \frac{8}{2}t^2 + 16t\right) + C$

$= \underline{\frac{7}{3}t^3 - 28t^2 + 112t + C}$.

(c) $\int \frac{3x^2+5}{4}\,dx = \frac{1}{4}\int 3x^2 + 5\,dx$

$= \frac{1}{4}\left(3\cdot\frac{x^3}{3} + 5x\right) + C$

$= \underline{\frac{1}{4}\left(x^3 + 5x\right) + C}$

(4).

(a) $\int_{-1}^{2} x\,dx = \left[\frac{1}{2}x^2\right]_{-1}^{2}$

$= \frac{1}{2}2^2 - \frac{1}{2}(-1)^2$

$= \frac{1}{2}\cdot 4 - \frac{1}{2}\cdot 1$

$= 2 - \frac{1}{2} = \underline{\underline{\frac{3}{2}}}.$

(b). $\int_0^1 x^2(x-3)\,dx = \int_0^1 x^3 - 3x^2\,dx$

$= \left[\frac{1}{4}x^4 - \frac{3x^3}{3}\right]_0^1$

$= \left[\frac{1}{4}x^4 - x^3\right]_0^1$

$= \left(\frac{1}{4}\cdot 1^4 - 1^3\right) - \left(\frac{1}{4}\cdot 0^4 - 0^3\right)$

$= \frac{1}{4} - 1 = \underline{\underline{\frac{-3}{4}}}.$