1M01 Calculus tutorial 2 solutions

1. (a) $f(x) = 2 - x^2$ defined for all real numbers x.

Reason: $x^2 = x \cdot x$ always def.

so $2 - x^2$ always def.

(b) $g(x) = x + \sqrt{x}$ defined only for $x \geq 0$.

Reason: $\sqrt{x}$ defined only for $x \geq 0$

So $x + \sqrt{x}$ ——— " ———.

(c) $h(x) = \dfrac{x-1}{x^2+4x}$ defined for all real numbers x except $x = 0$ & $x = -4$.

Reason: only possible issue is dividing by 0.

So/. When is $x^2 + 4x$ equal to 0?

$x^2 + 4x = 0$

a	1
b	4
c	0

Solutions:

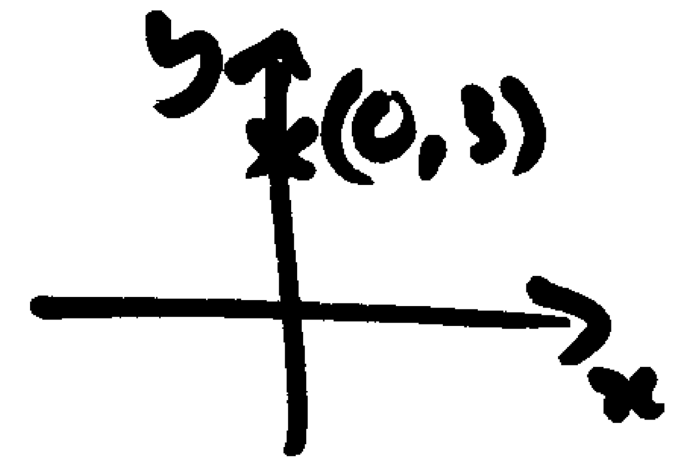
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-4 \pm \sqrt{16 - 0}}{2}$$

$$= \frac{-4+4}{2} \text{ or } \frac{-4-4}{2} = 0 \text{ or } -4.$$

(Or/: $x^2 + 4x = 0 \iff x(x+4) = 0 \iff x = 0$ or $x = -4$).

2. (a) Same slope as $y = -0.5x \rightsquigarrow$ slope $m = -0.5$

Passes through $(0,3)$



$\rightsquigarrow$ y-int is $c = 3$.

So equation is $y = mx + c$:

$$\underline{y = -0.5x + 3}$$

(b) Same slope as $y = 2x \rightsquigarrow$ slope $m = 2$.

Passes through $(x_1, y_1) = (6, 9)$

Use point-slope equation:

$$y - y_1 = m(x - x_1) \rightsquigarrow y - 9 = 2(x - 6)$$

Rearrange for slope-intercept equation:

$$y - 9 = 2(x-6) \Leftrightarrow y = 2x - 12 + 9$$

$$\Leftrightarrow \underline{y = 2x - 3}$$

(c) Passes thro' $(x_1, y_1) = (3, -4)$

& $(x_2, y_2) = (-1, 2)$

Slope is $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{2-(-4)}{-1-3} = \dfrac{6}{-4} = -1.5$

So point-slope equation is

$$y - y_1 = m(x - x_1): \quad y - (-4) = -1.5(x - 3)$$

$$\Leftrightarrow \underline{y + 4 = -1.5(x - 3)}$$

(or you can rearrange to get $\underline{y = -1.5x + 0.5}$ if you want).

3. Let x be skin surface area in m^2
Let y be brain mass in kg.

Then y directly proportional to x,
so $y = kx$ for some constant k.

Let's find k: for human,

x	1.7
y	1.3

$$1.3 = k \cdot 1.7, \text{ so}$$
$$k = \frac{1.3}{1.7}$$

⇝ General formula: $y = \frac{1.3}{1.7} x$

For a monkey with $x = 0.4$:

$$y = \frac{1.3}{1.7} \times 0.4 = \underline{0.3 \text{ kg}} \text{ (to 1 d.p.)}$$

4. $f(x) = -x^2 + 5x - 2$.

(a) All real numbers (because it's a polynomial)

(b) $f(x) = 0 \iff -x^2 + 5x + -2 = 0$

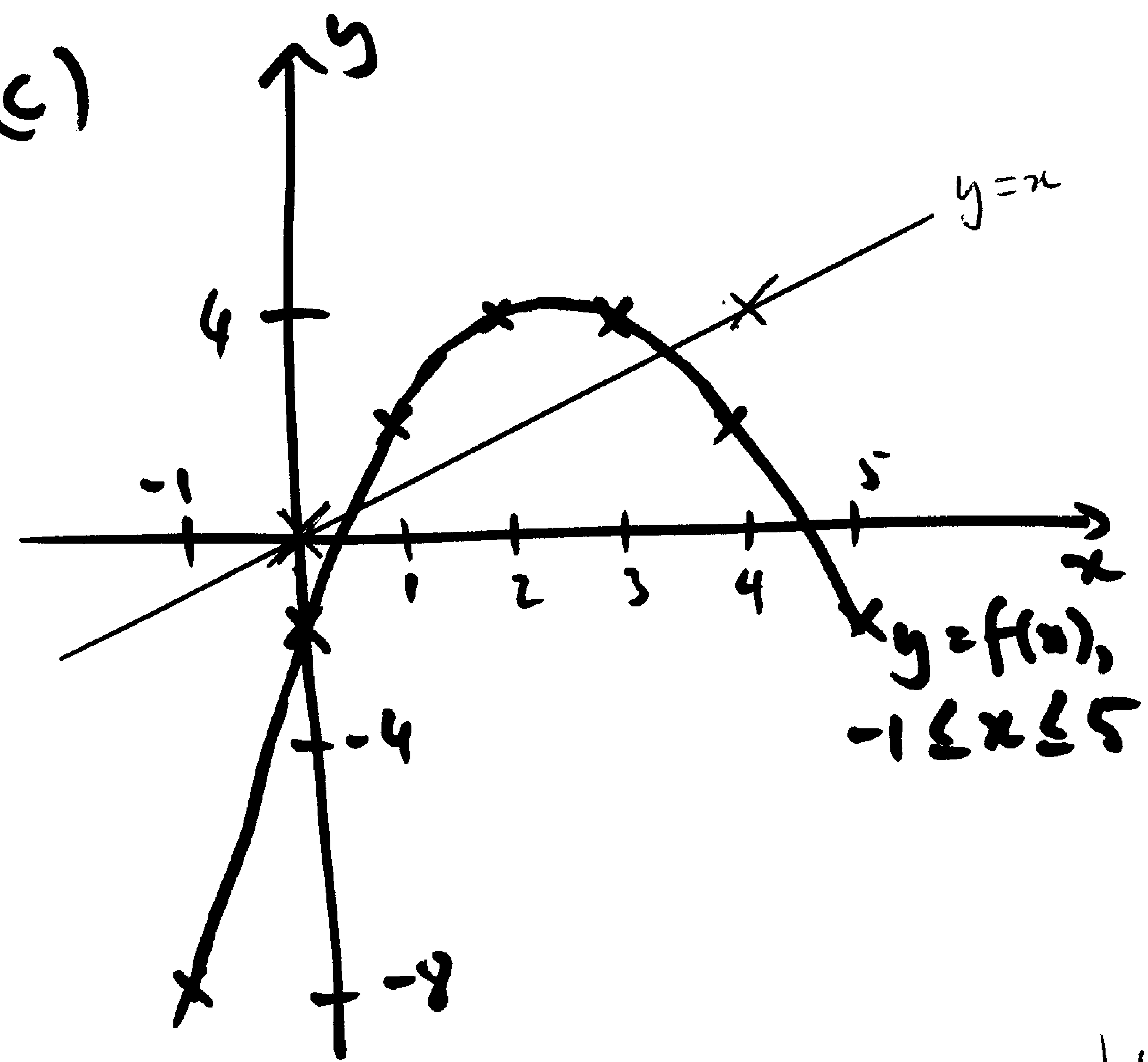
a	-1
b	5
c	-2

$$\iff x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-5 \pm \sqrt{25 - 4(-1)(-2)}}{2(-1)}$$

$$= \frac{-5 \pm \sqrt{25 - 8}}{-2} = \frac{5 \pm \sqrt{17}}{2}$$

$$= 4.56 \text{ or } 0.44 \quad \text{to 1 dp.}$$

(c)



x	$f(x)$
-1	-8
0	-2
1	2
2	4
3	4
4	2
5	-2

(d) $y = x$ hits the parabola at (roughly) $x = 0.5$ and $x = 3.3$

(based on my sketch; yours may give slightly different results).

Between those values, $y = f(x)$ is above $y = x$.

So $f(x) \geqslant x$ for $0.5 \leq x \leq 3.3$ (roughly)