

Maths methods 2010-11

Solutions to/tutorat exercise sheet 1.  
Calculus

$$\begin{aligned} \textcircled{1} \text{ (a)} \quad -6(y-2) + 4y &= -6y - 6 \cdot (-2) + 4y \\ &\quad \text{(by distributive law)} \\ &= -6y + 12 + 4y \\ &= \underline{\underline{-2y + 12}} \end{aligned}$$

$$\text{(b)} \quad 1.5 \underbrace{(-1)^2}_{=1} = \underline{\underline{1.5}}$$

$$\begin{aligned} \text{(c)} \quad -2(x-1)^2 \cdot (-2x) &= -2 \cdot (-2x) \cdot (x-1)^2 \\ &= 4x \cdot (x-1)^2 \\ &= 4x \cdot (x-1)(x-1) \\ &= 4x \cdot (x^2 - x - x + 1) \\ &= 4x(x^2 - 2x + 1) \\ &= \underline{\underline{4x^3 - 8x^2 + 4x}} \end{aligned}$$

$$\text{(d)} \quad (-0.1t)^3 = \underbrace{(-0.1)^3}_{=-0.001} \cdot t^3 = \underline{\underline{-0.001t^3}}$$

$$\begin{aligned} \text{(e)} \quad w^2 \cdot \frac{w \cdot w^{2n-3}}{(2w^n)^2} &= \frac{w^2 \cdot w^1 \cdot w^{2n-3}}{2^2 (w^n)^2} = \frac{w^{2+1+2n-3}}{4w^{2n}} \\ &= \frac{\cancel{w^{2n}}}{4w^{2n}} = \underline{\underline{\frac{1}{4}}} \end{aligned}$$

$$\textcircled{2} \quad (a) \quad \frac{1}{3} - \frac{1}{2} = \frac{2-3}{6} = \underline{\underline{\frac{-1}{6}}}$$

$$(b) \quad \frac{3x}{5} + \frac{1}{y^2} = \frac{3x \cdot y^2 + 5}{5y^2} = \underline{\underline{\frac{3xy^2 + 5}{5y^2}}}$$

$$\begin{aligned} (c) \quad (a+b)^{-1} - (a-b)^{-1} &= \frac{1}{a+b} - \frac{1}{a-b} \\ &= \frac{a-b - (a+b)}{(a+b)(a-b)} = \frac{a-b-a-b}{a^2-b^2} \\ &= \underline{\underline{\frac{-2b}{a^2-b^2}}} \quad \left( \text{or } \frac{-2b}{(a+b)(a-b)} \right) \end{aligned}$$

$$\textcircled{3} \quad (a) \quad \left(\frac{-2}{3}\right)^{-3} = \left(\frac{3}{-2}\right)^3 = \frac{3^3}{(-2)^3} = \frac{27}{-8} = \underline{\underline{-\frac{27}{8}}}$$

$$(b) \quad 4 \cdot 5^0 = 4 \times 1 = \underline{\underline{4}} \quad (\text{since } a^0 = 1 \text{ for any } a)$$

$$(c) \quad 2^{-3^2} = 2^{-(3^2)} = 2^{-9} = \frac{1}{2^9} = \underline{\underline{\frac{1}{512}}}$$

$$(d) \quad (x^2+4)^1 = x^2+4 \quad (\text{since } a^1 = a \text{ for any } a)$$

$$\begin{aligned} (e) \quad (3x)^2 (x^3)^{-4} &= 3^2 x^2 \cdot x^{-12} \\ &= 9x^2 \cdot \frac{1}{x^{12}} = \frac{9x^2}{x^{12}} = 9x^{2-12} \\ &= 9x^{-10} \\ &= \underline{\underline{\frac{9}{x^{10}}}} \end{aligned}$$

$$\begin{aligned}
 (4) \quad (a) \quad x + 4 = 3x &\Leftrightarrow 4 = 2x && (\text{subtract } x) \\
 &\Leftrightarrow 2 = x && (\div 2) \\
 &\Leftrightarrow \underline{\underline{x = 2}}.
 \end{aligned}$$

$$\begin{aligned}
 (b) \quad -\frac{2}{3}t = 8 &\Leftrightarrow \frac{2}{3}t = -8 && (\times -1) \\
 &\Leftrightarrow 2t = -24 && (\times 3) \\
 &\Leftrightarrow \underline{\underline{t = -12}} && (\div 2).
 \end{aligned}$$

$$\begin{aligned}
 (c) \quad 3(-1 + z^{-1}) = 4.5 \\
 \Leftrightarrow -1 + z^{-1} = 1.5 &&& (\div 3) \\
 \Leftrightarrow z^{-1} = \cancel{2.5} &&& (+1) \\
 \Leftrightarrow \frac{1}{z} = 2.5 &&& (\text{reciprocate LHS}) \\
 \Leftrightarrow \frac{1}{2.5} = z &&& \left( \begin{array}{l} \times z, \& \\ \div 2.5 \end{array} \right) \\
 \Leftrightarrow \underline{\underline{z = 0.4}} &&& \left( \frac{1}{2.5} = 0.4 \right).
 \end{aligned}$$