Towards braid-theoretic applications of Laver tables

Victoria LEBED
Joint work with Patrick DEHORNOY
Friday Seminar, July 18, 2014

Definition: Laver table $A_n$ is the set \{1, 2, 3, \ldots, 2^n\} endowed with the unique binary operation $\triangleright$ satisfying
\[ a \triangleright (b \triangleright c) = (a \triangleright b) \triangleright (a \triangleright c), \]
(Init)
\[ a \triangleright 1 \equiv a + 1 \mod 2^n. \]
(SD)

<table>
<thead>
<tr>
<th>$A_0$</th>
<th>$A_1$</th>
<th>$A_2$</th>
<th>$A_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
</tbody>
</table>

Figure 1: The first 5 Laver tables

\[ \pi_3(1) = 4 \]
\[ \pi_3(2) = 4 \]
\[ \pi_3(3) = 2 \]
\[ \pi_3(4) = 4 \]
\[ \pi_3(5) = 2 \]
\[ \pi_3(6) = 2 \]
\[ \pi_3(7) = 1 \]
\[ \pi_3(8) = 8 \]

Figure 2: Shelf colorings of a Reidemeister III move

\[ \phi(a, b) + \phi(a, c) + \phi(a \triangleright b, a \triangleright c) \]
\[ \phi(b, c) + \phi(a, b \triangleright c) + \phi(a, b) \]

Figure 3: Invariance of $\phi$-weight under RIII move

\[ \psi(a, b, c \triangleright d) + \psi(a \triangleright b, a \triangleright c, a \triangleright d) \]
\[ \psi(b, c, d) + \psi(a, b, c, b \triangleright d) \]

Figure 4: Invariance of $\psi$-weight under RIII move
Theorem (Dehornoy-L., ’14):

2) $\mathbb{Z}_2^n(A_n) \cong \mathbb{Z}^{2^n}$ basis: $\phi_{\text{const}}(a, b) = 1$ and coboundaries

$$\phi_{q,n}(a, b) = \begin{cases} 1 & \text{if } q \text{ occurs in the column } b, \\ 0 & \text{but not in the column } a \triangleright b \text{ of } A_n, \end{cases} \quad 1 \leq q < 2^n$$

3) $\mathbb{Z}_2^n(A_n) \cong \mathbb{Z}^{2^n-2^n+1}$ basis: $\psi_{\text{const}}(a, b, c) = 1$ and explicit $\{0, \pm 1\}$-valued coboundaries.

$$H \cong \mathbb{Z}_n(k \leq 3)$$

Theorem (L., ’14):

$k) \mathbb{Z}_2^k(A_n) \cong \mathbb{Z}^P_k(x^n)$. $P_k(x) = \frac{x^k + x^{\alpha(k)}}{x + 1}$, $\alpha(k) = \begin{cases} 1 & \text{if } k \text{ is even,} \\ 0 & \text{otherwise.} \end{cases}$

$$H \cong \mathbb{Z}_n(k \leq 3) \text{ for all } k.$$