Parameters and equations of cosmology[1]

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The **Robinson-Walker metric** is

$$ds^2 = -dt^2 + a(t)\left[\frac{1}{1-kr^2}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right] \tag{1}$$

where $k = 0$, 1 or -1. For $\Lambda = 0$ the equations are: the **Freidman equation**

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi\rho}{3} \tag{2}$$

the **acceleration equation**

$$\frac{\ddot{a}}{a} = -\frac{4\pi}{3}(\rho + 3p) \tag{3}$$

and the **fluid equation**

$$\dot{\rho} + \frac{3\dot{a}}{a}(\rho + p) = 0 \tag{4}$$

where p and ρ are the matter pressure and density. These are related by the equation of state, which depends on the type of matter. for dust, $p = 0$ and for radiation $p = \rho/3$.

It is common to related a and ρ to certain observationally significant parameters: the **Hubble parameter**

$$H = \frac{\dot{a}}{a} \tag{5}$$

the **deceleration parameter**

$$q = -\frac{\ddot{a}a}{\dot{a}^2} \tag{6}$$

and the **density parameter**

$$\Omega = \frac{8\pi}{3}\frac{\rho}{H^2} \tag{7}$$

and so, the **Freidman equation** is

$$1 + \frac{k}{a^2H^2} = \Omega \tag{8}$$

For $\Lambda \neq 0$ the equations are: the **Freidman equation**

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi\rho_M}{3} + \frac{\Lambda}{3} \tag{9}$$

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the **acceleration equation**

$$\frac{\ddot{a}}{a} = -\frac{4\pi}{3}(\rho_M + 3p_M) + \frac{\Lambda}{3} \tag{10}$$

with the fluid equation is unaffected. p_M and ρ_M are the matter pressure and density. These equations reduce the $\Lambda = 0$ equations above if we write $\rho = \rho_M + \rho_\Lambda$ and $p = p_M + p_\Lambda$ where $\rho_\Lambda = -p_\Lambda = \Lambda/8\pi$. We also define

$$\Omega_\Lambda = \frac{\Lambda}{3H^2} \tag{11}$$

and the **Freidman equation** becomes

$$1 + \frac{k}{a^2H^2} = \Omega_M + \Omega_\Lambda \tag{12}$$