442 Sample Paper - Outline answer to the first part of the first question[1]

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Define geodesic coordinates and writing the Riemann tensor in terms of these coordinates show $R_{abcd} = R_{cdab}$. [BOOK WORK]

Solution: If you use normal coördinates $\Gamma^c_{ab} = 0$, $g_{ab} = \eta_{ab}$ and the first derivatives of the metric are all zero. Basically, you loose the two terms which are second order in Γ. Substituting this back into the expression for the Riemann tensor it is easy to check that

$$R_{abcd} = \frac{1}{2}\left(-g_{ac,bd} + g_{ad,bc} + g_{bc,ad} - g_{bd,ac}\right) \tag{1}$$

so $R_{abcd} = R_{cdab}$ just follow from the symmetry of the metric and of the second derivative.

[1]Conor Houghton, houghton@maths.tcd.ie, see also http://www.maths.tcd.ie/~houghton/442.html