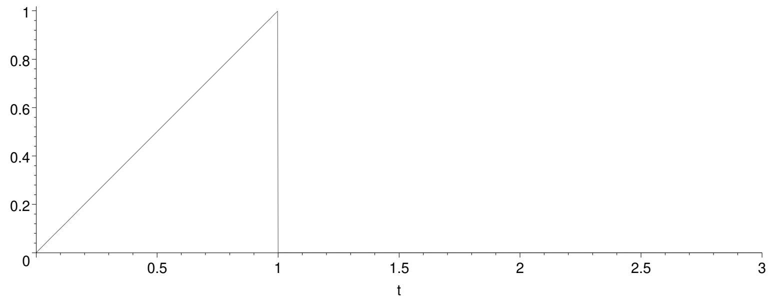
The Sawtooth function

9 November 2003

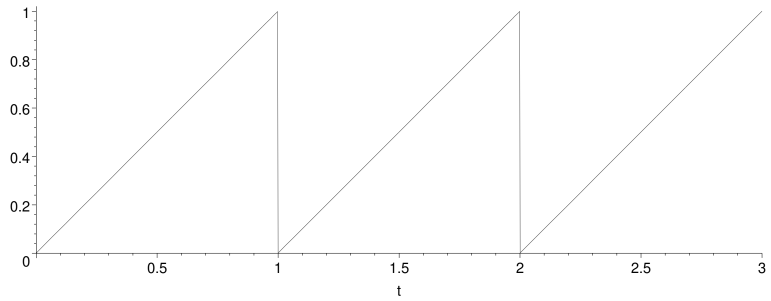
This used to be part of a problem sheet, this is why it is laid out in problem sheet format.

1. Sketch the two sawtooth and periodic sawtooth functions described in the next question.

   Solution: The sawtooth



   and the periodic sawtooth



2. Find the Laplace transform of the saw tooth function

$$f(t) = \begin{cases} t & 0 \leq t < 1 \\ 0 & t \geq 1 \end{cases} \tag{1}$$

   Next, find the Laplace transform of the periodic saw tooth function with period one given by

$$\begin{aligned} f(t) &= t \qquad 0 \leq t < 1 \\ f(t+1) &= f(t) \end{aligned} \tag{2}$$

Solution: From the definition of the Laplace transform and integrating by parts

$$\begin{aligned}
\mathcal{L}(f) &= \int_0^\infty f(t)e^{-st}\,dt = \int_0^1 te^{-st}\,dt \\
&= \left[-\frac{t}{s}e^{-st}\right]_{t=0}^1 + \int_{t=0}^1 \frac{1}{s}e^{-st}\,dt \\
&= -\frac{1}{s}e^{-sc} + 0 + \left[\frac{1}{-s^2}e^{-st}\right]_{t=0}^{t=1} \\
&= \frac{1}{s^2} - \frac{1}{s}e^{-s} - \frac{1}{s^2}e^{-s}
\end{aligned}$$

We know the Laplace transform of the periodic function is

$$\mathcal{L}(f) = \frac{1}{1-e^{-s}}\int_0^1 f(t)e^{-s}\,dt \tag{3}$$

and this integral is identical to the one we just did in the previous question. So the answer for the periodic saw tooth is

$$\frac{1}{1-e^{-s}}\left(\frac{1}{s^2} - \frac{1}{s}e^{-s} - \frac{1}{s^2}e^{-s}\right) \tag{4}$$