2E2 Tutorial Sheet 6 First Term

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1. (2) Find the Z-transform of the sequence \((1, 0, 2, -3, 0, 0, \ldots)\).

Solution: So, use the formula
\[
Z[(x_n)] = \sum_{n=0}^{\infty} x_n z^{-n}
\]

(1)
to get
\[
Z[(1, 0, 2, -3, 0, 0, \ldots)] = 1 + 2z^{-2} - 3z^{-3}
\]

(2)

2. (2) Find the Z-transform of the geometric sequence \((3, 6, 12, 24, \ldots)\).

Solution: This sequence has the form \(3 \times 2^n\) so we can use the formula for the geometric sequence to get
\[
Z[(3 \times 2^n)] = 3Z[(2^n)] = \frac{3z}{z - 2}
\]

(3)

3. (2) Find the Z-transform of the delayed sequence \((0, 0, 0, 3, 6, 12, 24, \ldots)\).

Solution: So the delay formula says that
\[
Z[(x_{k-k_0})] = \frac{1}{z^{k_0}}Z[(x_k)]
\]

(4)

In this example, the sequence is delayed by three steps, \(k_0 = 3\) and apart from that it is the same as the above example, so
\[
Z[(0, 0, 0, 3, 6, 12, 24, \ldots)] = \frac{1}{z^3} \frac{3z}{z - 2} = \frac{3}{z^3(z - 2)}
\]

(5)

4. (2) Find the Z-transform of the sequence \((6, 12, 24, \ldots)\) both by considering it the advance of the sequence \((3, 6, 12, 24, \ldots)\) and by applying the formula for geometrical sequences directly. Do you get the same answer?

Solution: Now this example is advanced by one step, so we use the formula
\[
Z[(x_{k+1})] = zX(z) - zx_0
\]

(6)

where \(X(z) = Z[(x_n)]\). In this case we have
\[
Z[(3, 6, 12, 24, \ldots)] = \frac{3z}{z - 2}
\]

(7)

so
\[
Z[(6, 12, 24, \ldots)] = z \frac{3z}{z - 2} - 3z = \frac{3z^2}{z - 2} - 3z = \frac{6z}{z - 2}
\]

(8)

Working directly, this is the sequence \(6 \times 2^n\) and so the Z-transform is
\[
Z[(6 \times 2^n)] = 6Z[(2^n)]\frac{6}{z - 2}
\]

(9)

which is, of course, the same answer.

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