2E2 Tutorial Sheet 6 First Term[1]

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1. (2) Find the Z-transform of the sequence $(1, 0, 2, -3, 0, 0, \ldots)$.

   Solution: So, use the formula

$$\mathcal{Z}[(x_n)] = \sum_{n=0}^{\infty} \frac{x_n}{z^n} \tag{1}$$

   to get

$$\mathcal{Z}[(1, 0, 2, -3, 0, 0, \ldots)] = 1 + \frac{2}{z^2} - \frac{3}{z^3} \tag{2}$$

2. (2) Find the Z-transform of the geometric sequence $(3, 6, 12, 24, \ldots)$.

   Solution: This sequence has the form 3×2^n so we can use the formula for the geometric sequence to get

$$\mathcal{Z}[(3 \times 2^n)] = 3\mathcal{Z}[(2^n)] = \frac{3z}{z-2} \tag{3}$$

3. (2) Find the Z-transform of the delayed sequence $(0, 0, 0, 3, 6, 12, 24, \ldots)$

   Solution: So the delay formula says that

$$\mathcal{Z}[(x_{k-k_0})] = \frac{1}{z^{k_0}} \mathcal{Z}[(x_k)] \tag{4}$$

   In this example, the sequence is delayed by three steps, $k_0 = 3$ and apart from that it is the same as the above example, so

$$\mathcal{Z}[(0, 0, 0, 3, 6, 12, 24, \ldots)] = \frac{1}{z^3} \frac{3z}{z-2} = \frac{3}{z^2(z-2)} \tag{5}$$

4. (2) Find the Z-transform of the sequence $(6, 12, 24, \ldots)$ both by considering it the advance of the sequence $(3, 6, 12, 24, \ldots)$ and by applying the formula for geometrical sequences directly. Do you get the same answer?

   Solution: Now this example is advanced by one step, so we use the formula

$$\mathcal{Z}[(x_{k+1})] = zX(z) - zx_0 \tag{6}$$

   where $X(z) = \mathcal{Z}[(x_n)]$. In this case we have

$$\mathcal{Z}[(3, 6, 12, 24, \ldots)] = \frac{3z}{z-2} \tag{7}$$

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   so

$$\mathcal{Z}[(6, 12, 24, \ldots)] = z\frac{3z}{z-2} - 3z = \frac{3z^2}{z-2} - 3z = \frac{6z}{z-2} \tag{8}$$

   Working directly, this is the sequence 6×2^n and so the Z-transform is

$$\mathcal{Z}[(6 \times 2^n)] = 6\mathcal{Z}[(2^n)] \frac{6}{z-2} \tag{9}$$

   which is, of course, the same answer.