**2E2 Tutorial Sheet 6 First Term**[1]

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1. (2) Find the Z-transform of the sequence $(1, 0, 2, -3, 0, 0, \ldots)$.

   *Solution:* So, use the formula
$$\mathcal{Z}[(x_n)] = \sum_{n=0}^{\infty} \frac{x_n}{z^n} \tag{1}$$
   to get
$$\mathcal{Z}[(1, 0, 2, -3, 0, 0, \ldots)] = 1 + \frac{2}{z^2} - \frac{3}{z^3} \tag{2}$$

2. (2) Find the Z-transform of the geometric sequence $(3, 6, 12, 24, \ldots)$.

   *Solution:* This sequence has the form $3 \times 2^n$ so we can use the formula for the geometric sequence to get
$$\mathcal{Z}[(3 \times 2^n)] = 3\mathcal{Z}[(2^n)] = \frac{3z}{z-2} \tag{3}$$

3. (2) Find the Z-transform of the delayed sequence $(0, 0, 0, 3, 6, 12, 24, \ldots)$

   *Solution:* So the delay formula says that
$$\mathcal{Z}[(x_{k-k_0})] = \frac{1}{z^{k_0}} \mathcal{Z}[(x_k)] \tag{4}$$
   In this example, the sequence is delayed by three steps, $k_0 = 3$ and apart from that it is the same as the above example, so
$$\mathcal{Z}[(0, 0, 0, 3, 6, 12, 24, \ldots)] = \frac{1}{z^3} \frac{3z}{z-2} = \frac{3}{z^2(z-2)} \tag{5}$$

4. (2) Find the Z-transform of the sequence $(6, 12, 24, \ldots)$ both by considering it the advance of the sequence $(3, 6, 12, 24, \ldots)$ and by applying the formula for geometrical sequences directly. Do you get the same answer?

   *Solution:* Now this example is advanced by one step, so we use the formula
$$\mathcal{Z}[(x_{k+1})] = zX(z) - zx_0 \tag{6}$$
   where $X(z) = \mathcal{Z}[(x_n)]$. In this case we have
$$\mathcal{Z}[(3, 6, 12, 24, \ldots)] = \frac{3z}{z-2} \tag{7}$$

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so

$$\mathcal{Z}[(6,12,24,\ldots)] = z\frac{3z}{z-2} - 3z = \frac{3z^2}{z-2} - 3z = \frac{6z}{z-2} \tag{8}$$

Working directly, this is the sequence $6 \times 2^n$ and so the Z-transform is

$$\mathcal{Z}[(6 \times 2^n)] = 6\mathcal{Z}[(2^n)]\frac{6}{z-2} \tag{9}$$

which is, of course, the same answer.