1. (2) Using the linearity of the Laplace transform, calculate the Laplace transform of 

\[ f(t) = \sinh(at) = \frac{e^{at} - e^{-at}}{2} \]

**Solution:** Well, just write it out

\[ \mathcal{L}(\sinh(at)) = \mathcal{L}\left(\frac{e^{at} - e^{-at}}{2}\right) = \frac{1}{2} \mathcal{L}(e^{at}) - \frac{1}{2} \mathcal{L}(e^{-at}) \]

\[ = \frac{1}{2} \left( \frac{1}{s-a} - \frac{1}{s+a} \right) = \frac{a}{s^2 - a^2} \]  

(1)

2. (2) Using the shift theorem find the Laplace transform of

\[ f(t) = e^{2t}t^2 \]

**Solution:** Recall the first shift theorem says

\[ \mathcal{L}\left(e^{at}f(t)\right) = F(s-a) \]

where \( \mathcal{L}(f) = F(s) \). Now, we know that

\[ \mathcal{L}\left(t^n\right) = \frac{n!}{s^{n+1}} \]

so, by the shift theorem

\[ \mathcal{L}\left(e^{2t}t^2\right) = \frac{2}{(s-2)^3} \]  

(4)

3. (2) Find the Laplace transform of both side of the identity

\[ \frac{d}{dt} \cosh 3t = 3 \sinh 3t \]

4. (2) Find the Laplace transform of both sides of the differential equation

\[ \frac{2}{s^2} \frac{df}{dt} = 1 \]

with initial conditions \( f(0) = 4 \). By solving the resulting equations find \( F(s) \). Based on the Laplace transforms you know, decide what \( f(t) \) is.

**Solution:** Using linearity of \( \mathcal{L} \), plus the property of Laplace transforms of derivatives we get

\[ \mathcal{L}\left(\frac{2ds}{dt}\right) = \mathcal{L}(1) \]

\[ 2\mathcal{L}\left(\frac{df}{dt}\right) = \frac{1}{s} \]

\[ 2sF(s) - 8 = \frac{1}{s} \]  

(7)

This means that

\[ F(s) = \frac{4}{s} + \frac{1}{2s^2} \]  

(5)

and, since \( \mathcal{L}(t^n) = \frac{n!}{s^{n+1}} \)

\[ f = 4 + \frac{1}{2}t \]  

To verify that this solves the equation note that \( f(0) = 4 \) as required and \( f' = 1/2 \)