which means
\[ l = \int_0^1 2t\sqrt{1 + \pi^2} \, dt = \sqrt{1 + \pi^2} t^2 \bigg|_0^1 = \sqrt{1 + \pi^2} \]

**Aside** An actual picture of the curve is given in Figure 1. Notice that the parameter \( t \) plays a hidden role, it tells you how fast \( r(t) \) moves along the curve as \( t \) changes.

The curve \( s(t) = \sin \pi t + \cos \pi t \mathbf{j} + 2k \) is the same as a curve in space, but different values of \( t \) correspond to different points. For example \( s(\sqrt{2}) = \mathbf{j} + 2k \) so the point \((0, 1, 2)\) is on the curve and corresponds to \( t = \sqrt{2} \). However, \( s \) is the same curve and the point \((0, 1, 2)\) is on it, but corresponds to \( t = 2 \).

3. The figure shows the curve \( r(t) = \cos^2 t \mathbf{i} + \sin^3 t \mathbf{j} \). Work out its total length.

**Solution:** So, here the curve is closed and so we need to decide a starting point. For \( t = 0 \), \( r = \mathbf{i} \) and if \( r(t) = \mathbf{i} \) this implies that \( t = 0 \) or \( 2\pi \) and so on. Thus, we can find the total length by integrating from \( t_1 = 0 \) and \( t_2 = 2\pi \). Now, by differentiating
\[
 r'(t) = 3 \sin t \cos^2 t \mathbf{i} + 2 \cos t \sin^2 t \mathbf{j}
\]
and so \( |r'| = \pm 3 \sqrt{(\sin t \cos^2 t)^2 + (\cos t \sin^2 t)^2} = 3 |\sin t \cos t| \). The slightly confusing point is that the length of \( r' \) is always positive so you have to take the positive or negative square root according to which gives a positive answer. Now, using the periodicity of the sine
\[
\frac{3}{2} \int_0^{2\pi} |\sin 2t| \, dt = 6 \int_0^{\pi/2} \sin 2t \, dt
\]
and so the length is
\[
6 \int_0^{\pi/2} \sin 2t \, dt = 3 \cos 2t \bigg|_0^{\pi/2} = 6
\]