2E2 Tutorial Sheet 8 First Term[1]

28 November 2003

1. (3) Find the eigenvectors and eigenvalues of the following matrices

$$(i)\quad \begin{pmatrix} 4 & 0 \\ 0 & -6 \end{pmatrix} \qquad (ii)\quad \begin{pmatrix} 10 & -4 \\ 18 & -12 \end{pmatrix} \qquad (iii)\quad \begin{pmatrix} 0 & r \\ r & 0 \end{pmatrix} \tag{1}$$

2. (2) Rewrite

$$\begin{aligned} 10y_1 - 4y_2 &= 2 \\ 18y_1 - 12y_2 &= 3 \end{aligned} \tag{2}$$

in the matrix form $A\mathbf{y} = \mathbf{a}$ where

$$\mathbf{y} = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \tag{3}$$

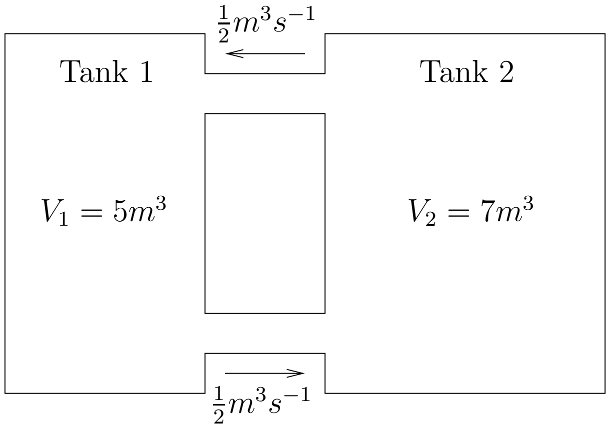

Figure 1: Two containers with flow between them.

3. (3) As illustrated in Fig. 1, two large containers are connected and American style sandwich spead is pumped between them at a rate of $1/2m^3s^{-1}$. One container has volume $5m^3$, the other $7m^3$. Both are full of spread. Initially the smaller container contains pure jam, the second container has $5m^3$ of jam and $2m^3$ of peanut butter. Assume perfect mixing and so on.
(i) Write down the differential equation for $y_1(t)$ and $y_2(t)$, the amount of peanut butter in the first and second container.
(ii) Solve it to find $y_1(t)$ and $y_2(t)$ explicitly.
(iii) Use the initial data to find the values of the constants in the solution.

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