## 28 November 2003

1. (3) Find the eigenvectors and eigenvalues of the following matrices

$$(i) \begin{pmatrix} 4 & 0 \\ 0 & -6 \end{pmatrix} \qquad (ii) \begin{pmatrix} 10 & -4 \\ 18 & -12 \end{pmatrix} \qquad (iii) \begin{pmatrix} 0 & r \\ r & 0 \end{pmatrix} \qquad (1)$$

2. (2) Rewrite

$$\begin{array}{rcl}
10y_1 - 4y_2 &=& 2\\
18y_1 - 12y_2 &=& 3
\end{array} \tag{2}$$

in the matrix form  $A\mathbf{y} = \mathbf{a}$  where

$$\mathbf{y} = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \tag{3}$$



Figure 1: Two containers with flow between them.

3. (3) As illustrated in Fig. 1, two large containers are connected and American style sandwich spead is pumped between them at a rate of  $1/2m^3s^{-1}$ . One container has volume  $5m^3$ , the other  $7m^3$ . Both are full of spread. Initially the smaller container contains pure jam, the second container has  $5m^3$  of jam and  $2m^3$  of peanut butter. Assume perfect mixing and so on.

(i) Write down the differential equation for  $y_1(t)$  and  $y_2(t)$ , the amount of peanut butter in the first and second container.

- (ii) Solve it to find  $y_1(t)$  and  $y_2(t)$  explicitly.
- (iii) Use the initial data to find the values of the constants in the solution.

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