The Gauss divergence theorem relates certain three dimensional integrals to surface integral. It states that

\[ \int \int \int_V \text{div} \mathbf{F} \, dV = \int \int_S \mathbf{F} \cdot \mathbf{n} \, dA \]  

where \( V \) is some three dimensional region, \( S \) is its boundary and \( \mathbf{n} \) is an outward pointing unit normal.

1. (2) Use the Gauss divergence theorem to calculate

\[ \int \int_S \mathbf{F} \cdot \mathbf{n} \, dA \]

where \( \mathbf{F} = (3x, 3y, -9z) \) and \( S \) is the surface of the cylinder between the two discs of radius one, perpendicular to the \( z \)-axis and with centers at \((0, 0, -1)\) and \((0, 0, 2)\).

2. (2) Use the Gauss divergence theorem to calculate

\[ \int \int_S \mathbf{F} \cdot \mathbf{n} \, dA \]

where \( \mathbf{F} = (3x^2 + z, 3y^2, -9yz) \) and \( S \) is the surface of the cube with unit edges and vertices at \((0,0,0)\), \((1,0,0)\), \((0,1,0)\), \((0,0,1)\), \((1,1,0)\), \((1,0,1)\), \((0,1,1)\) and \((1,1,1)\).

3. (2) Integrate

\[ \int \int_S (yi + zxj) \cdot \mathbf{n} \, dA \]

where \( S \) is the surface of a sphere of radius one whose center is the origin.

4. (2) What is the outward pointing unit normal to the unit sphere, centered on the origin? What is \( \text{div} \mathbf{F} \) where \( \mathbf{F} = (x/r, y/r, z/r) \)? Now, using the Gauss divergence theorem, calculate

\[ \int \int \int_V \frac{1}{r} \, dV \]

where \( V \) is the unit sphere of radius one centered on the origin.

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