2E2 Tutorial Sheet third term 22[1]

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The Gauss divergence theorem relates certain three dimensional integrals to surface integral. It states that

$$\int\int\int_V \operatorname{div}\mathbf{F}\,dV = \int\int_S \mathbf{F}\cdot\mathbf{n}\,dA \tag{1}$$

where V is some three dimensional region, S is its boundary and $\mathbf{n}$ is an outward pointing unit normal.

1. (2) Use the Gauss divergence theorem to calculate

$$\int\int_S \mathbf{F}\cdot\mathbf{n}\,dA \tag{2}$$

where $\mathbf{F} = (3x, 3y, -9z)$ and S is the surface of the cylinder between the two discs of radius one, perpendicular to the z-axis and with centers at $(0, 0, -1)$ and $(0, 0, 2)$.

2. (2) Use the Gauss divergence theorem to calculate

$$\int\int_S \mathbf{F}\cdot\mathbf{n}\,dA \tag{3}$$

where $\mathbf{F} = (3x^2 + z, 3y^2, -9zy)$ and S is the surface of the cube with unit edges and vertices at $(0, 0, 0)$, $(1, 0, 0)$, $(0, 1, 0)$, $(0, 0, 1)$, $(1, 1, 0)$, $(1, 0, 1)$, $(0, 1, 1)$ and $(1, 1, 1)$.

3. (2) Integrate

$$\int\int_S (y\mathbf{i} + zx\mathbf{j})\cdot\mathbf{n}dA \tag{4}$$

where S is the surface of a sphere of radius one whose center is the origin.

4. (2) What is the outward pointing unit normal to the unit sphere, centered on the origin? What is $\operatorname{div}\mathbf{F}$ where $\mathbf{F} = (x/r, y/r, z/r)$? Now, using the Gauss divergence theorem, calculate

$$\int\int\int_V \frac{1}{r}\,dV \tag{5}$$

where V is the unit sphere of radius one centered on the origin.

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