2E2 Tutorial Sheet 21 Third Term[1]

20 April 2004

1. (1) Find a unit normal to $z = \sqrt{x^2 + y^2}$ at $(6, 8, 10)$. Note that this surface is $f = 0$ where $f = z - \sqrt{x^2 + y^2}$
2. (2) What is the div of
   (a) $v_1(y, z)\mathbf{i} + v_2(x, z)\mathbf{j} + v_3(x, y)\mathbf{k}$
   (b) $xyz(x\mathbf{i} + y\mathbf{j} + z\mathbf{k})$
   (c) $(x^2 + y^2 + z^2)^{-3/2}(x\mathbf{i} + y\mathbf{j} + z\mathbf{k})$
3. (2) What is the curl of
   (a) $v_1(x)\mathbf{i} + v_2(y)\mathbf{j} + v_3(z)\mathbf{k}$
   (b) $xyz(x\mathbf{i} + y\mathbf{j} + z\mathbf{k})$
   (c) $(x^2 + y^2 + z^2)^{-3/2}(x\mathbf{i} + y\mathbf{j} + z\mathbf{k})$
4. (1) Show $\nabla \times (\nabla \mathbf{f}) = 0$.
5. (2) Show $\text{curl}(f\mathbf{v}) = (\text{grad}\, f) \times \mathbf{v} + f\text{curl}\, \mathbf{v}$.

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