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**Vectors calculus.** With \( f(x, y, z) \) a scalar field the grad of \( f \), \( \nabla f \), is

\[
\text{grad } f = \frac{\partial f}{\partial x} \mathbf{i} + \frac{\partial f}{\partial y} \mathbf{j} + \frac{\partial f}{\partial z} \mathbf{k}
\]  

(1)

1. (2) \( f = x^2 + 2x^2yz \), find grad \( f \). If \( g = x^2yz + 2z \), what is \( \nabla f \cdot \nabla g \).

2. (2) Find the directional derivative of \( z = (x^2 + y^2) \) in the direction \( \mathbf{i} \).

3. (2) \( f = xyz \), work out grad \( f \). What is the value of grad \( f \) at \((2, 1, 2)\). What is the directional derivative of \( f \) in the \( \mathbf{i} \)-direction at \((2, 1, 2)\).

4. (2) \( f = x^2 + y^2 + z^2 \), find grad \( f \). What is the directional derivative of \( f \) in the direction of \( \mathbf{b} = (1, 1, 1) \) at the point \((1, 1, 1)\). Remember to use a unit vector when working out the directional derivative. What is the directional derivative in the \( \mathbf{i} \) direction at \((1, 0, 0)\); what about the \( \mathbf{j} \) direction?

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