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1. (3) Assuming the solution of

\[(1 - x)y' + y = 0\]  \hspace{1cm} (1)

has a series expansion about \(x = 0\) work out the recursion relation. Write out the first few terms and show that the series terminates to give \(y = A(1 - x)\) for arbitrary \(A\).

2. (3) Assuming the solution of

\[(1 - x^2)y' - 2xy = 0\]  \hspace{1cm} (2)

has a series expansion about \(x = 0\), work out the recursion relation and write out the first four non-zero terms.

3. (2) Assuming the solution of

\[y'' - 3y' + 2y = 0\]  \hspace{1cm} (3)

has a series expansion about \(x = 0\), by substitution, work out the recursion relation. If \(y(0) = 1\) and \(y'(0) = 0\) what are the first three non-zero terms.

**Note:** If

\[y = \sum_{n=0}^{\infty} a_n x^n\]  \hspace{1cm} (4)

then, by setting \(x = 0\)

\[y(0) = a_0.\]  \hspace{1cm} (5)

Similarly,

\[y' = \sum_{n=0}^{\infty} na_n x^{n-1}\]  \hspace{1cm} (6)

and, by setting \(x = 0\)

\[y'(0) = a_1.\]  \hspace{1cm} (7)

On the other hand if no initial condition is given then for a first order equation \(a_0\) is arbitrary and for a second order equation \(a_0\) and \(a_1\) are both arbitrary, so when you write out the non-zero terms, there are \(a_0\)'s and \(a_1\)'s appearing. In Q1 the arbitrary constant \(a_0\) has been renamed \(A\), this is just to make the solution look nicer.

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