2E2 Tutorial Sheet 16 Second Term[1]

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1. (3) Assuming the solution of
$$(1-x)y' + y = 0 \tag{1}$$
has a series expansion about $x = 0$ work out the recursion relation. Write out the first few terms and show that the series terminates to give $y = A(1-x)$ for arbitrary A.

2. (3) Assuming the solution of
$$(1-x^2)y' - 2xy = 0 \tag{2}$$
has a series expansion about $x = 0$, work out the recursion relation and write out the first four non-zero terms.

3. (2) Assuming the solution of
$$y'' - 3y' + 2y = 0 \tag{3}$$
has a series expansion about $x = 0$, by substitution, work out the recursion relation. If $y(0) = 1$ and $y'(0) = 0$ what are the first three non-zero terms.

Note: If
$$y = \sum_{n=0}^{\infty} a_n x^n \tag{4}$$
then, by setting $x = 0$
$$y(0) = a_0. \tag{5}$$
Similarily,
$$y' = \sum_{n=0}^{\infty} n a_n x^{n-1} \tag{6}$$
and, by setting $x = 0$
$$y'(0) = a_1. \tag{7}$$

On the other hand if no initial condition is given then for a first order equation a_0 is arbitrary and for a second order equation a_0 and a_1 are both arbitrary, so when you write out the non-zero terms, there are a_0's and a_1's appearing. In Q1 the arbitrary constant a_0 has been renamed A, this is just to make the solution look nicer.

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