2E2 Tutorial Sheet 10 Second Term[1]

9 January 2003

1. (2) Last week you were asked to find the solution for the system

$$\begin{aligned}\frac{dy_1}{dt} &= -3y_1 + 2y_2 \\ \frac{dy_2}{dt} &= -2y_1 + 2y_2\end{aligned}$$

   The solution is

$$\mathbf{y} = c_1 \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{t} + c_2 \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{-2t} \tag{1}$$

   Now draw the phase diagram for this solution and name the type of stationary point (saddlepoint, or outward improper node.)

2. (2) Last week you were asked to find the solution for the system

$$\frac{dy_1}{dt} = 3y_1 + y_2 \tag{2}$$

$$\frac{dy_2}{dt} = y_1 + 3y_2 \tag{3}$$

   The solution is

$$\mathbf{y} = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{4t} + c_2 \begin{pmatrix} -1 \\ 1 \end{pmatrix} e^{2t}. \tag{4}$$

   Sketch the phase diagram and and describe the stationary point.

3. (4) Find the general solutions for the system

$$\frac{dy_1}{dt} = 2y_1 - y_2 \tag{5}$$

$$\frac{dy_2}{dt} = -4y_2 \tag{6}$$

   Sketch the phase diagram and and describe the stationary point.

[1] Conor Houghton, houghton@maths.tcd.ie and http://www.maths.tcd.ie/~houghton/ 2E2.html