Christmas quiz with Answers[1]

10 December 2001

1. What are the Carlow colours? Red and Green, or at least, this is what I thought, someone has written to me with the following: *Question 1 on Christmas quiz 2001 your answer isn't totally right. Your missing the third Carlow colour of gold (or yellow) depending on who you ask but still your wrong.* So, sorry about that.

2. Find the Laplace transform of the function which is zero from zero to one, one from one to two, minus one from two to four and then zero from then on.

   Well you can answer this by direct integration, lets call the function f:

$$f = \begin{cases} 0 & 0 \leq t < 1 \\ 1 & 1 \leq t < 2 \\ -1 & 2 \leq t < 4 \\ 0 & t \geq 4 \end{cases} \tag{1}$$

   the

$$F(s) = \int_0^\infty f(t)e^{-ts}dt = \int_1^2 e^{-ts}dt - \int_2^4 e^{-ts}dt = \frac{e^{-s} - 2e^{-2s} + e^{-4s}}{s} \tag{2}$$

   The other way to do it would be to write f in terms of two Heaviside step functions.

3. Solve $y'' + 3y' + 2y = \delta(t-2)$ with $y(0) = y'(0) = 0$.

   This is like PS6 Q1, if we take the Laplace transform of both sides we get

$$(s^2 + 3s + 2)Y(s) = e^{-2s} \tag{3}$$

   so

$$Y(s) = \left(-\frac{1}{s+2} + \frac{1}{s+1}\right)e^{-2s} \tag{4}$$

   Now,

$$\mathcal{L}\left(-e^{-2t} + e^{-t}\right) = -\frac{1}{s+2} + \frac{1}{s+1} \tag{5}$$

   the delay theorem tells us that

$$y = H_2(t)\left(-e^{-2t+4} + e^{-t+2}\right) \tag{6}$$

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4. Name two of these three films: (i) 'time to die', (twice), (ii) 'shut up and deal', (iii) 'I don't know what it is about saxaphone players': (i) Blade Runner, (ii) The Apartment, (iii) Some Like it Hot.

5. Calculate $t * e^{2t}$. This is PS 5 Q2.

6. What is the next term in this sequence: 14, 23,28, 34, 42, 50, 59, 66, 72, 79, 86, 96, 103, ?. Cathedral Parkway, they are stops on the number 9 subway in New York city.

7. If
$$Y(z) = \frac{2z+1}{(z+1)(z-3)}$$
what is y_2?

   Well, by partial fractions
$$Y(z) = \frac{1}{4(z+1)} + \frac{7}{4(z-3)} \tag{7}$$
So, using the shifting theorem, $y_0 = 0$ and for $k > 1$
$$y_k = \frac{1}{4}(-1)^{k-1} + \frac{7}{4}3^{k-1} \tag{8}$$
Hence $y_2 = 5$.

8. Whose epitaph is $e^{i\pi} = -1$? Leonard Euler, actually I have no evidence that this is true it is just something someone told me once. It is certainly true that Euler was the first to use π to mean what it now means, sometime in the mid C18.

9. Solve $y_{k+2} + y_{k+1} - 2y_k = (-2)^k$ with $y_0 = y_1 = 0$.

   Take the Z transform of both sides
$$(z^2 + z - 2)Y(z) = \frac{z}{z+2} \tag{9}$$
so, by partial fractions,
$$Y(z) = -\frac{z}{3(z+2)^2} - \frac{z}{9(z+2)} + \frac{z}{9(z-1)} \tag{10}$$
so
$$y_k = -\frac{1}{3}k(-2)^k - \frac{1}{9}(-2)^k + \frac{1}{9}. \tag{11}$$

10. This year the end of Ramadan is in December, name the last day of Ramadan, the last day of Advent and the Jewish festival which is also this month. Ide or Id-al-Fitr (more correctly this is the three day celebration following the end of Ramadan), Christmas ends Advent and Chanukah or Hannaka is the Jewish festival.