A convolution example[1]

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This short example can be done either using the convolution theorem or using partial fractions. Say

$$F = \frac{6}{s(s+3)} \tag{1}$$

let us try and calculate $f9t)$ both by partial fractions and by convolutions.

First, partial fractions, let

$$\frac{1}{s(s+3)} = \frac{A}{s} + \frac{B}{s+3} \tag{2}$$

then

$$1 = A(s+3) + Bs \tag{3}$$

and choosing $s = 0$ gives $A = 1/3$. Choosing $s = -3$ gives $B = -1/3$. This means that

$$F = \frac{2}{s} - \frac{2}{s+3} \tag{4}$$

and, so,

$$f = 2 - 2e^{-3t} \tag{5}$$

By the convolution theorem,

$$F = \frac{6}{s(s+3)} = \frac{6}{s}\frac{1}{s+3} = \mathcal{L}(6)\mathcal{L}\left(e^{-3t}\right) = \mathcal{L}\left(6 * e^{-3t}\right) \tag{6}$$

using $\mathcal{L}(f)\mathcal{L}(g) = \mathcal{L}(f*g)$. So, we need to work out $6 * \exp(-3t)$. Remember the formula for the convolution.

$$f * g = \int_0^t f(\tau)g(t-\tau)d\tau \tag{7}$$

In this case, it doesn't make much difference, but we can use $f*g = g*f$ and work out $\exp(-3t) * 6$ instead of $6 * \exp(-3t)$, it is a tiny bit easier:

$$\exp(-3t) * 6 = 6\int_0^t \exp(-3\tau)d\tau = -2\exp(-3\tau)]_0^t = 2 - 2e^{-3t} \tag{8}$$

as before.

[1] Conor Houghton, houghton@maths.tcd.ie, see also http://www.maths.tcd.ie/~houghton/2E2.html