Wavelets and LRD time series

David Malone

<dwmalone@{maths.tcd,cnri.dit}.ie>

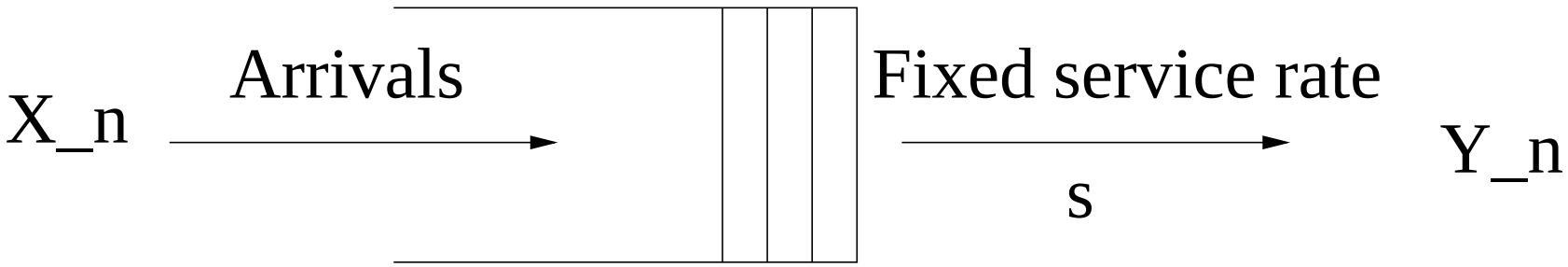
October 2004

Outline

- Outline of network problem.
- Outline of wavelet method.
- What we discovered.
- What about wavelet method was useful.

Work with Ken Duffy (CNRI) and Chris King (Northeastern).

Simple Network



$$Q_n = \max(X_n + Q_{n-1} - s, 0) \quad (1)$$

$$Y_n = \min(X_n + Q_{n-1}, s) \quad (2)$$

Interested in waiting times and queue lengths.

Depends on input process.

Long Range Dependence

Network traffic known to have interesting statistics. Exhibits LRD features. (Stationary) Process Y_n is LRD if:

$$\sum_{k \in \mathbb{Z}} |\rho(k)| = \infty, \tag{3}$$

where ρ is the autocorrelation. Alternatively,

$$\lim_{\theta \to 0} S(\theta) \sim \theta^{-\beta} \tag{4}$$

where S is the Power Spectrum ($\hat{\rho}$), and $\beta \in (0, 1)$.

Wavelet Estimator

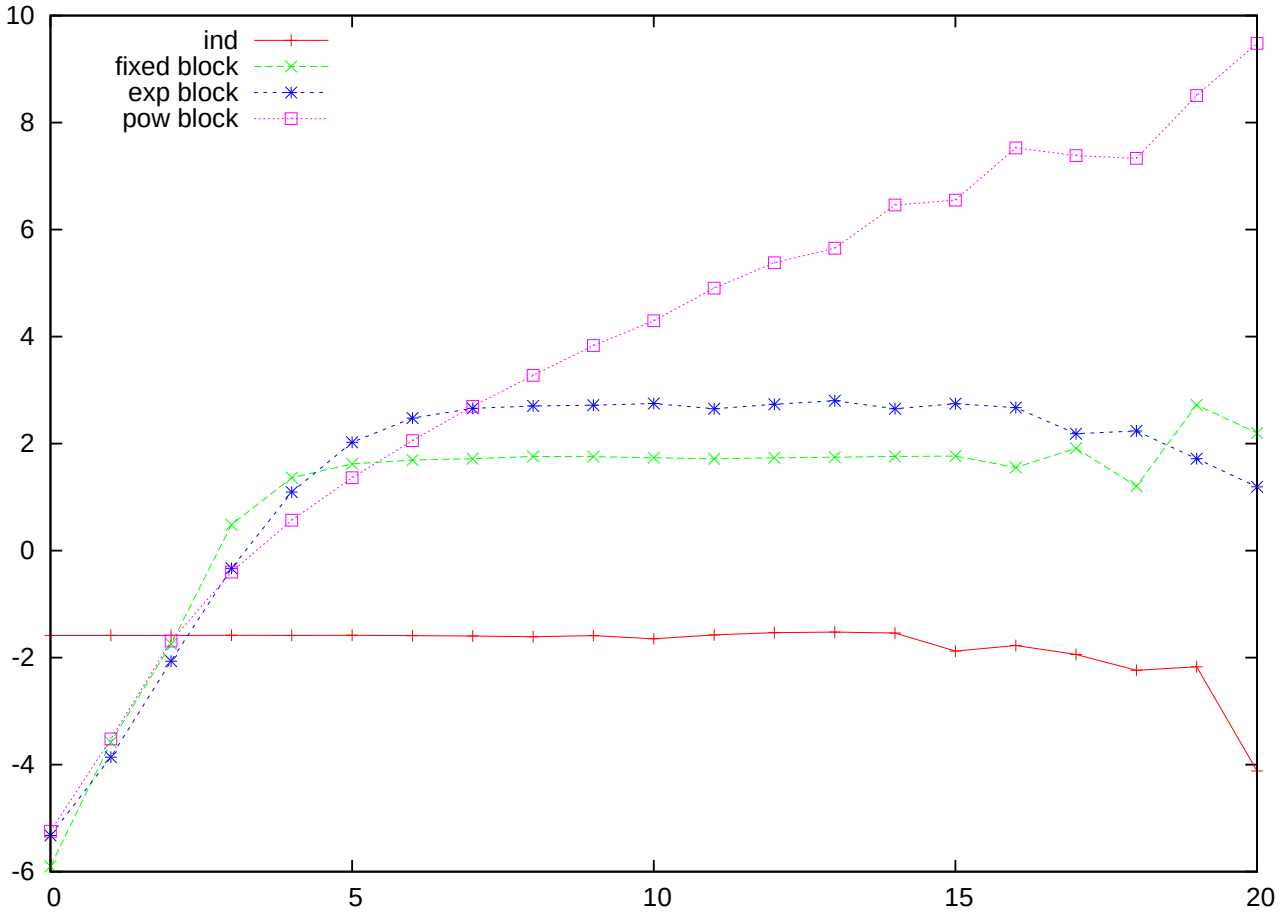
The wavelet coefficients $d(j,k)$ can be used to estimate the power spectrum at particular points:

$$S(2^{-j}\theta_0) \sim \frac{1}{n_j} \sum_k |d(j,k)|^2 \tag{5}$$

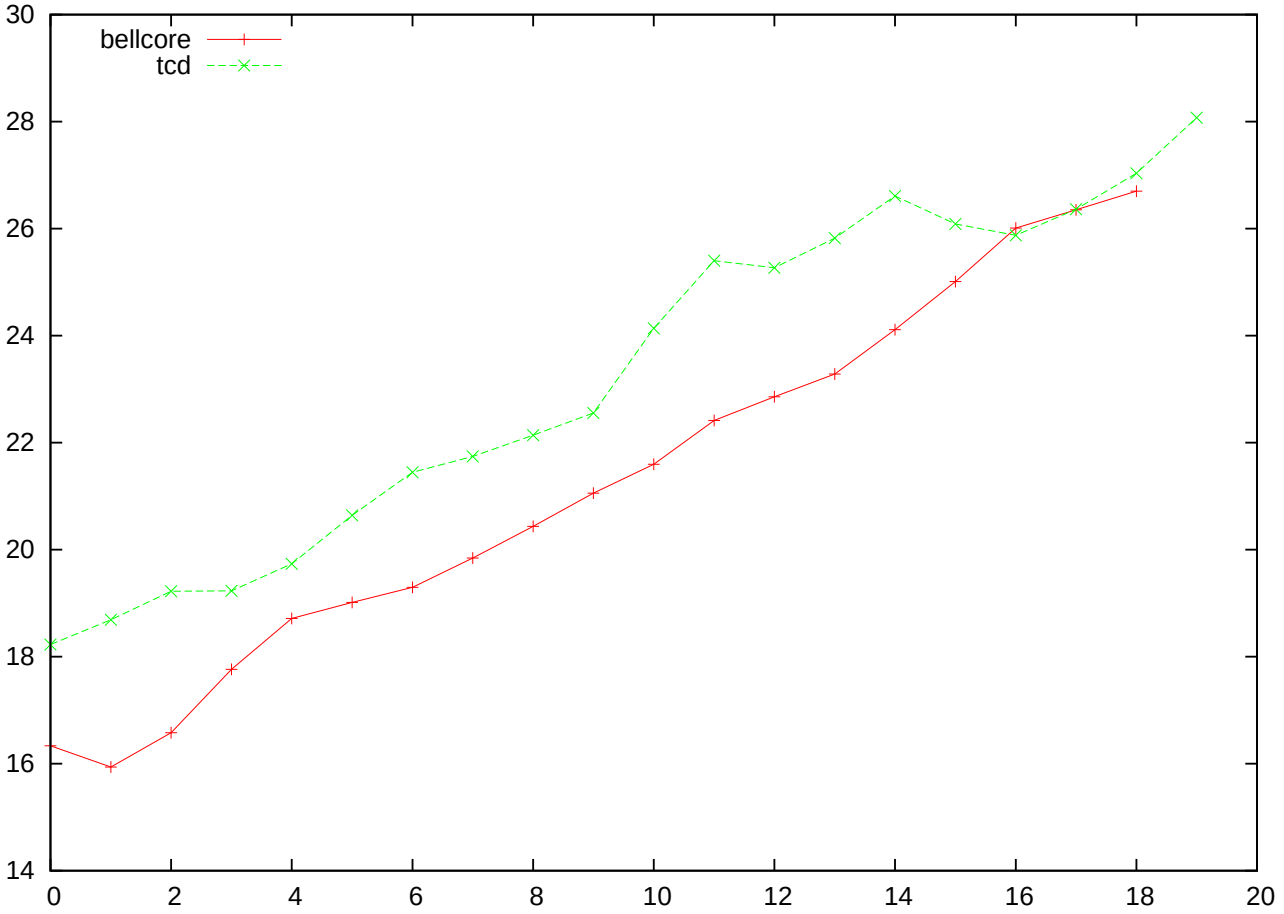
where n_j is the number of coefficients at scale j. (Abry et al.) Can plot $\log(1/n_j \sum |d(j,k)|^2)$ against j and estimate slope.

Has desirable properties wrt: bias, convergence, speed (FWT), ignores trends, ...

Example estimates

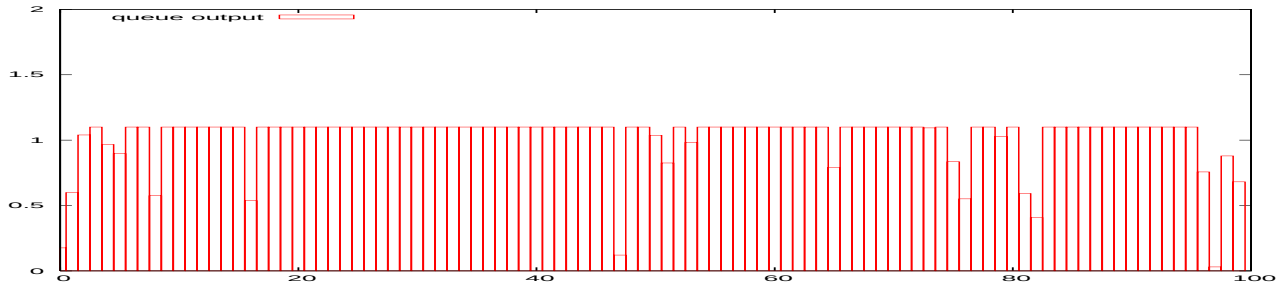


Real estimates

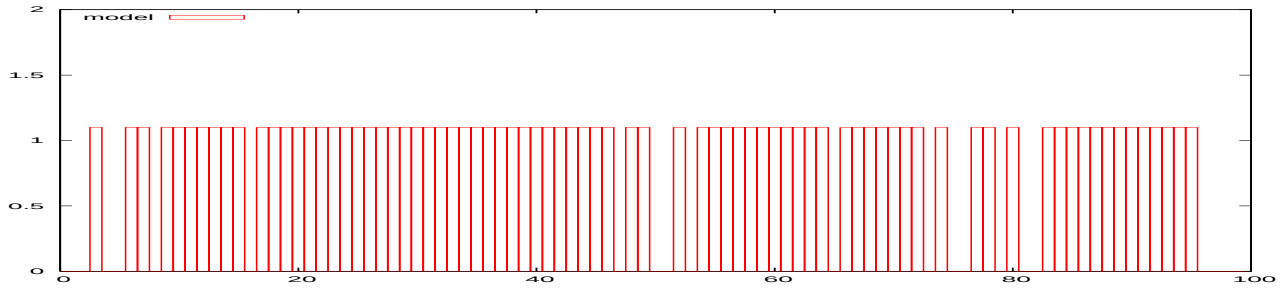


On/Off Queues

Queue output:



Model with On/Off traffic:



On periods IID, Off periods IID.

LRD and On/Off

LRD from On/Off traffic $\Rightarrow$ heavy tails.

Typically things like

$$\mathbb{P}\left[B_{\text{On}} \geq x\right] \sim x^{-\alpha} \tag{6}$$

for $\alpha \in (1, 2)$.

The Hurst parameter $H = (3 - \alpha)/2$.

Heavy tail queues

How do we get heavy tails from queues?

For a stable queue, putting SRD in $\Rightarrow$ SRD out.

Have to put LRD in to get LRD out. Eg, Zwart 2002

$$\mathbb{P}[B > x] \sim x^{-\nu}, \nu > 1 \Rightarrow \mathbb{P}[P > x] \sim \mathbb{P}[B > x] \qquad (7)$$

Overloaded queues are always on.

Critical case

In balanced case, queue becomes like random walk with no drift.

$$\mathbb{E}[B^2] < \infty \Rightarrow \mathbb{P}[P > x] \sim x^{-1/2} \tag{8}$$

Even stranger.

$$\mathbb{P}[B > x] \sim x^{-\nu}, 1 < \nu < 2 \Rightarrow \mathbb{P}[P > x] \sim x^{-1/\nu} \tag{9}$$

LRD from SRD via balanced queueing?

Pull other LRD?

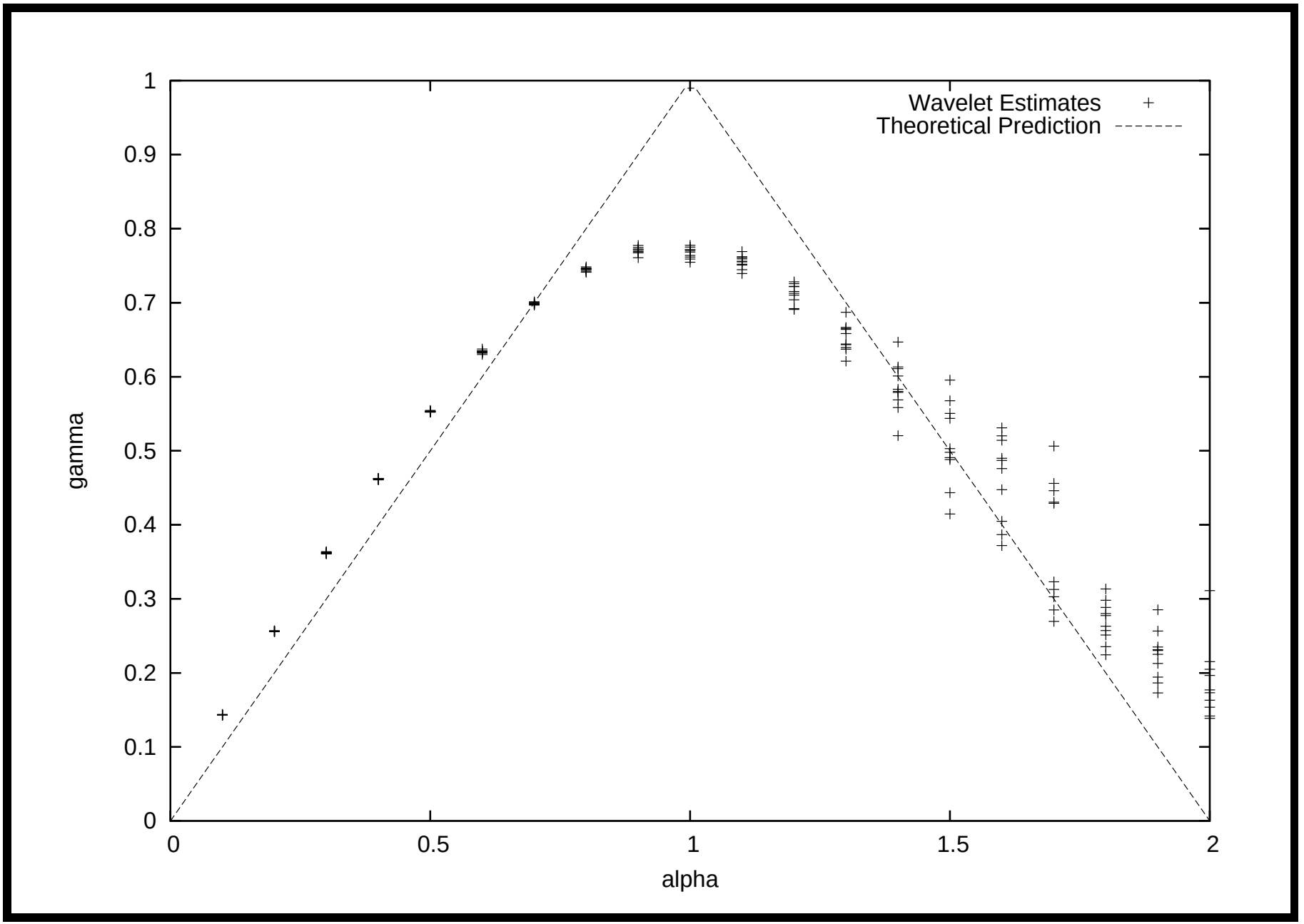
Problem: isn't going to be stationary.

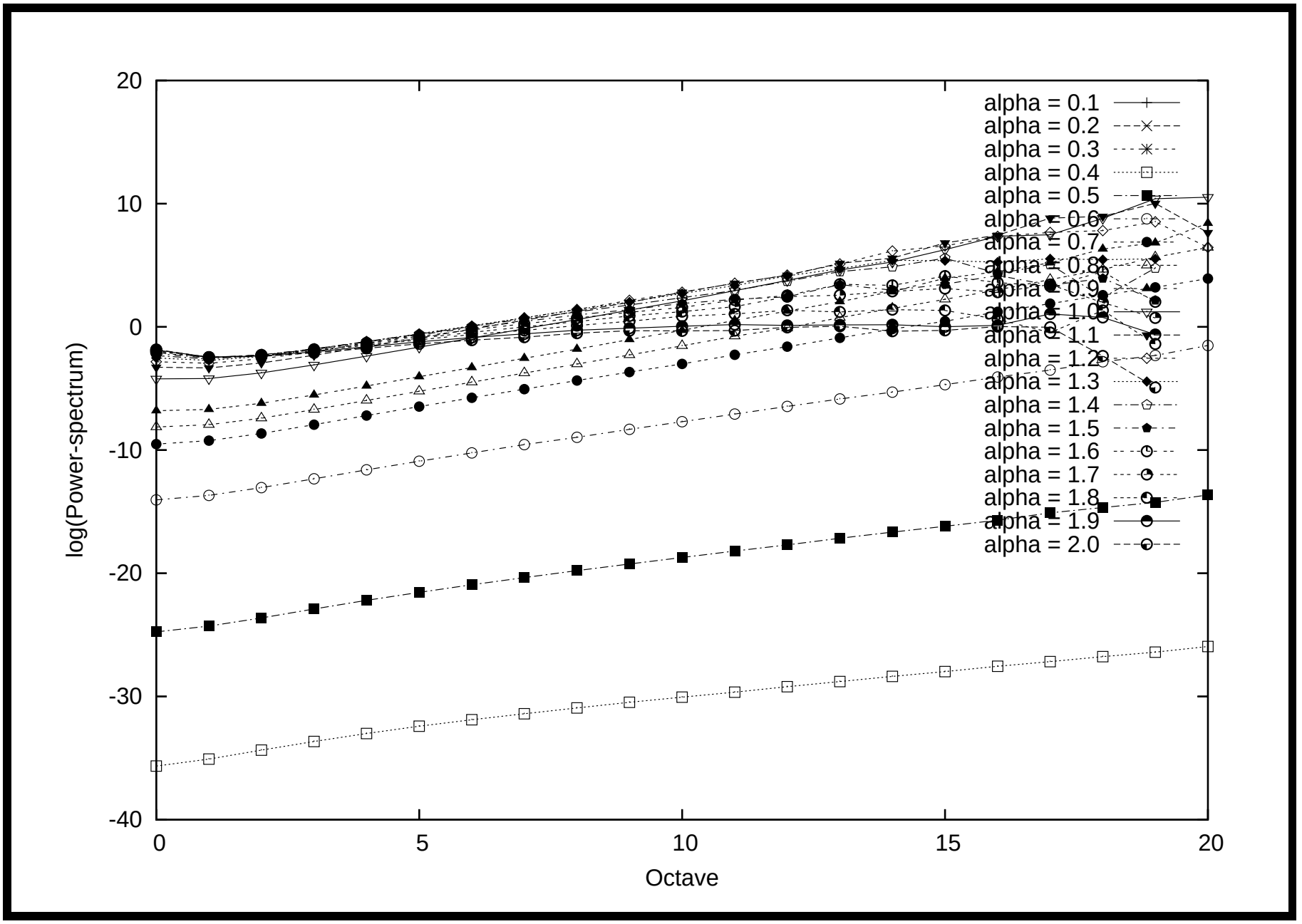
Problem: power spectrum is ill-defined.

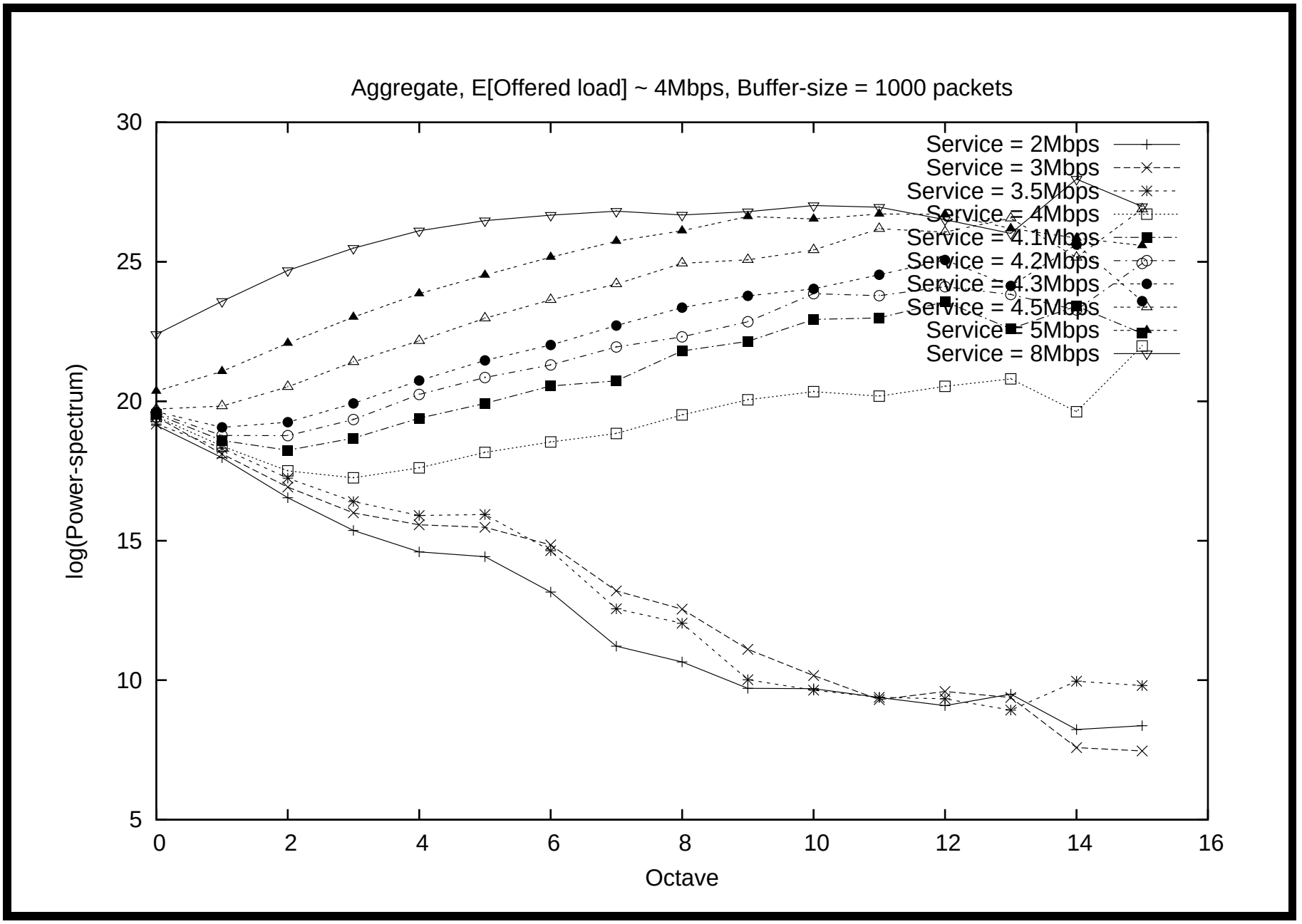
$$\tilde{X}_\epsilon(\theta) = \int_0^\infty X(t) e^{(2\pi i\theta - \epsilon)t} dt \tag{10}$$

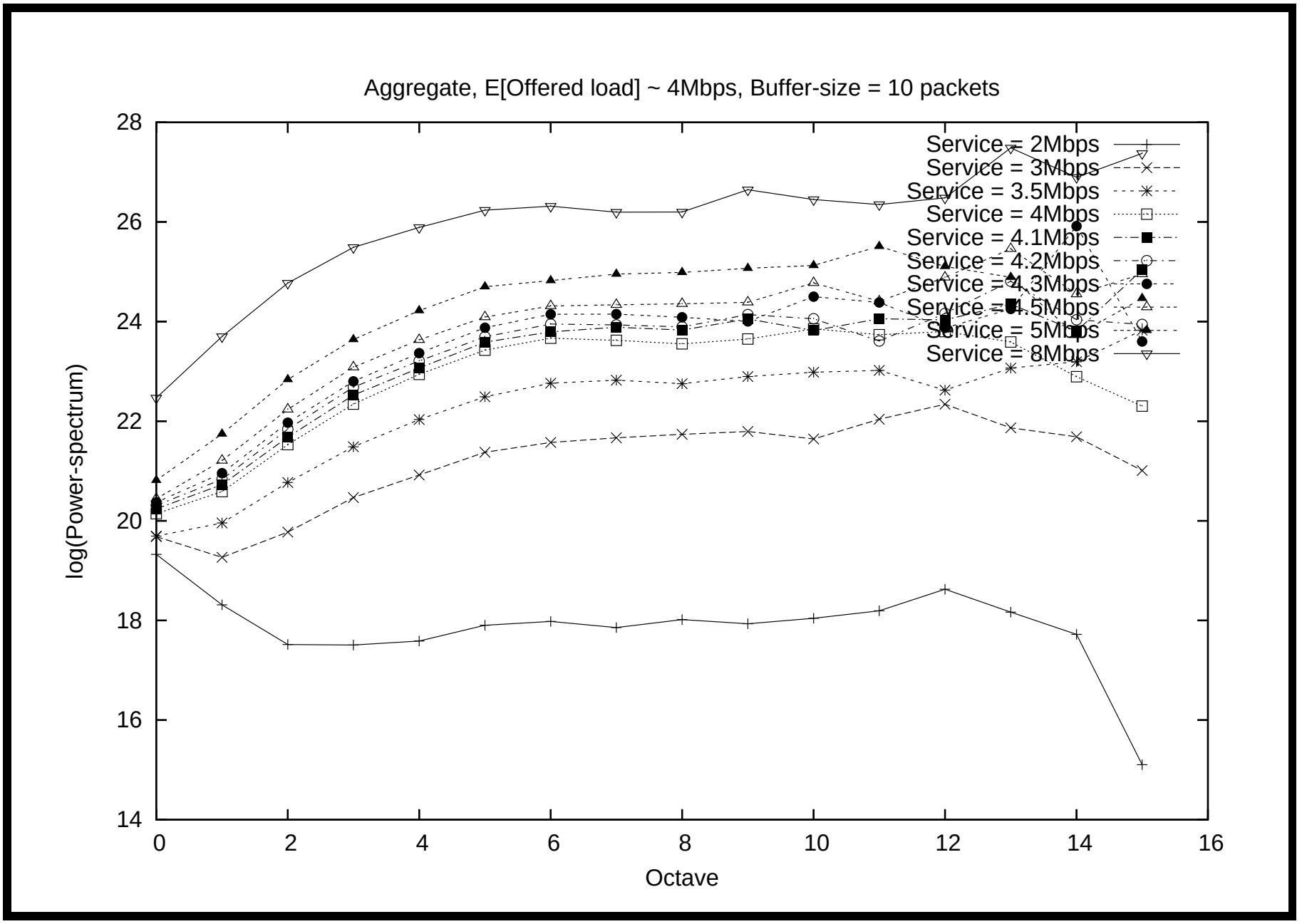
$$S^{\text{reg}}(\theta) = \lim_{\epsilon \to 0} \ \epsilon^\mu \ \mathbb{E}\Big[|\tilde{X}_\epsilon(\theta)|^2 \Big] \tag{11}$$

where $\mu = 1$ if $\nu > 1$ and $\mu = \nu$ for $0 < \nu < 1$.


Wavelet Estimates
Theoretical Prediction
gamma
alpha
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
0.5
1
1.5
2


log(Power-spectrum)
Octave
20
10
0
-10
-20
-30
-40
0
5
10
15
20
alpha = 0.1
alpha = 0.2
alpha = 0.3
alpha = 0.4
alpha = 0.5
alpha = 0.6
alpha = 0.7
alpha = 0.8
alpha = 0.9
alpha = 1.0
alpha = 1.1
alpha = 1.2
alpha = 1.3
alpha = 1.4
alpha = 1.5
alpha = 1.6
alpha = 1.7
alpha = 1.8
alpha = 1.9
alpha = 2.0


Aggregate, E[Offered load] ~ 4Mbps, Buffer-size = 1000 packets
log(Power-spectrum)
Octave
30
25
20
15
10
5
0
2
4
6
8
10
12
14
16
Service = 2Mbps
Service = 3Mbps
Service = 3.5Mbps
Service = 4Mbps
Service = 4.1Mbps
Service = 4.2Mbps
Service = 4.3Mbps
Service = 4.5Mbps
Service = 5Mbps
Service = 8Mbps


Aggregate, E[Offered load] ~ 4Mbps, Buffer-size = 10 packets
log(Power-spectrum)
Octave
28
26
24
22
20
18
16
14
0
2
4
6
8
10
12
14
16
Service = 2Mbps
Service = 3Mbps
Service = 3.5Mbps
Service = 4Mbps
Service = 4.1Mbps
Service = 4.2Mbps
Service = 4.3Mbps
Service = 4.5Mbps
Service = 5Mbps
Service = 8Mbps

Conclusions

- Wavelet estimator of LRD relatively well behaved.
- Fast for running data through.
- Particularly good for long bursts.
- Interesting when applied outside normal range.