Measuring Time

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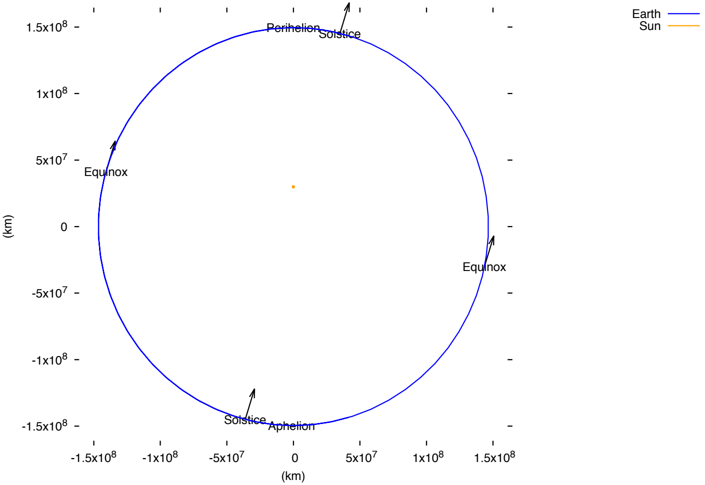
Calendar Year

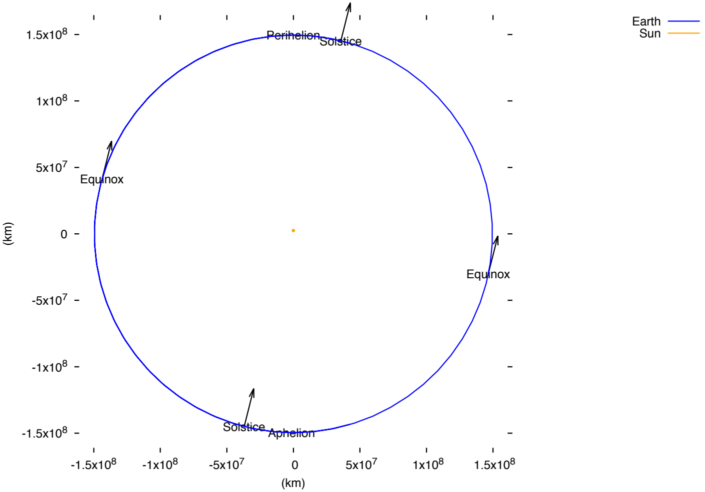
Seasons: Weather cycles, days lengthen and shorten.

Aim of our calendar: Keep Equinoxes and Solstices at the right time of year, especially the vernal equinox.
Tricky: year isn't whole number of days (365.24219).

The time of year: angle between earth's axis and the line from the earth to the sun.

NB: seasons nothing to do with distance to sun. Earth is at its closest (Perihelion) about 4$^{\text{th}}$ January 2017.


Earth
Sun
Perihelion
Solstice
Equinox
Equinox
Solstice
Aphelion
1.5x10^8
1x10^8
5x10^7
0
-5x10^7
-1x10^8
-1.5x10^8
-1.5x10^8
-1x10^8
-5x10^7
0
5x10^7
1x10^8
1.5x10^8
(km)
(km)


Earth
Sun
Perihelion
Solstice
Equinox
Equinox
Solstice
Aphelion
1.5x10^8
1x10^8
5x10^7
0
-5x10^7
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(km)
-1.5x10^8
-1x10^8
-5x10^7
0
5x10^7
1x10^8
1.5x10^8
(km)

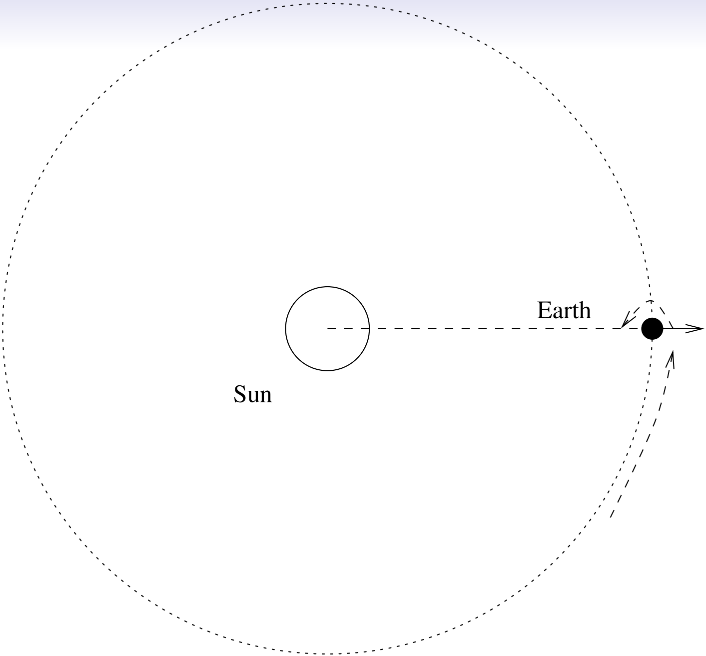
Days

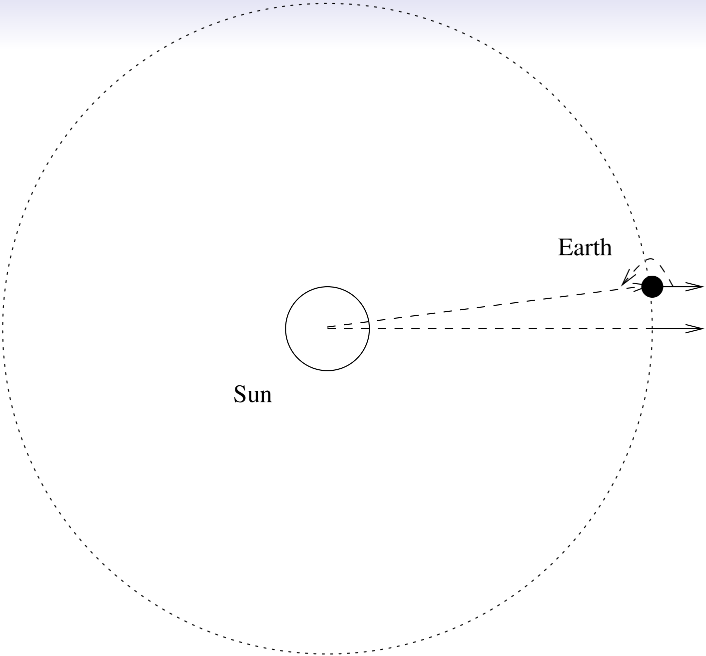
Obviously, it gets dark and bright once per day!

Different cultures start days at: sunset, sunrise, midnight, midday, ...

Must be something to do with Earth going around.

Solar vs. sidereal days.


Earth
Sun


Earth
Sun

Hours

Arbitrary divisions of a day. They arise by dividing things into 12.

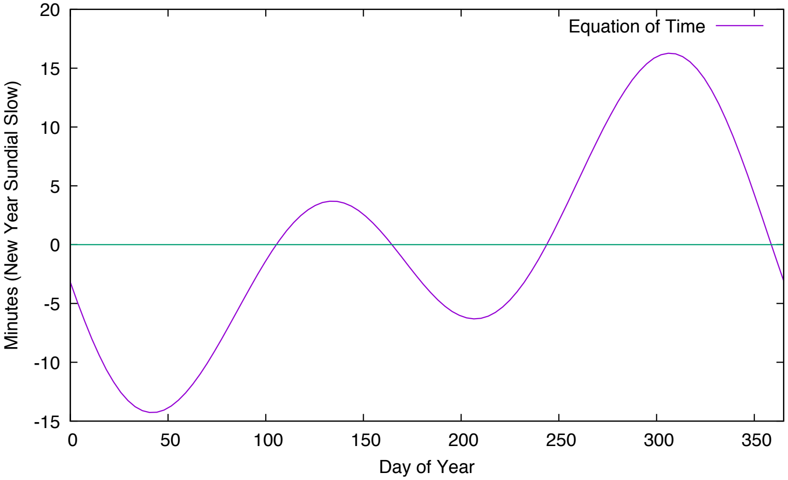
Were very uneven. Gradually fixed (14C).

Came to us via monastery and Roman army.

In 7C, lots of subdivisions, by middle ages we have *minutae primae* and *minutae secondae*.

Apparent vs. Mean Time

In 1792, move from apparent time to mean time.



Problem with midnight — it depends where you live.

Dunsink Observatory

Usher: 25m7–48s (1787).
Brinkley: 25m22s (1832).
Romney-Robinson: 25m21s (1838).
Elliott-(Ray-Drury-Malone): 25m21.02s (2017).

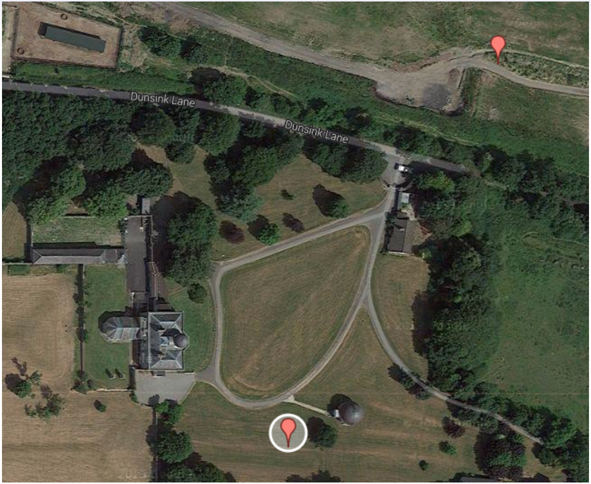


Image: Google Maps

$6°20.3', 53°23.2'$ vs $6°20.2', 53°23.3'$

Atomic Seconds

International Atomic Time has been available since 1955 (officially since 1972). Uses SI second.

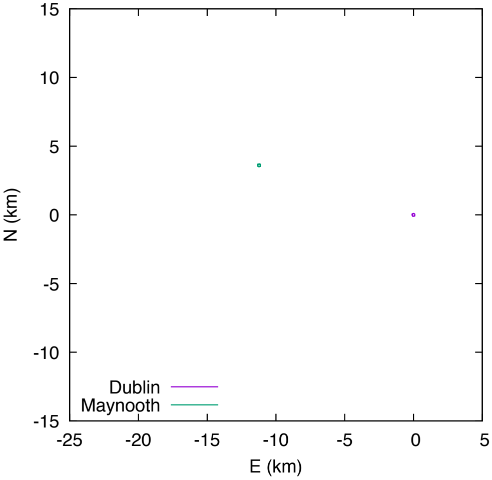
second: In the International System of Units (SI), the time interval equal to 9,192,631,770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium-133 atom.

How did they pick 9,192,631,770?
Used ET, based on Newcomb's measurements from 1750–1892.

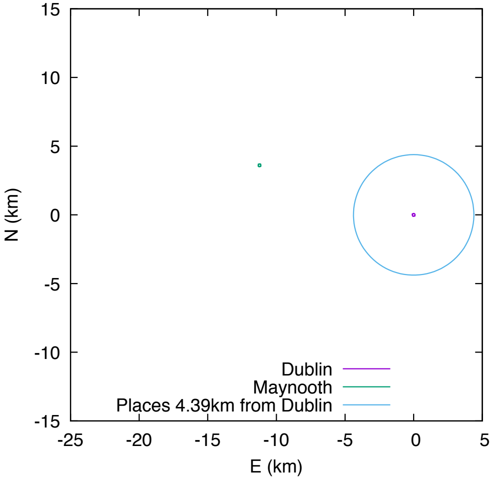
GPS

- GPS is Global Positioning system.
- Example of a GNSS (global navigation satellite system).
- Run by US, originally for military.
- Basically a bunch of flying atomic clocks!
- Carefully adjusted to keep the same time, despite *time dilation*.
- They transmit the time.
- Frequency of transmission gets Doppler shifted.
- They also transmit an *almanac* saying where the satellites are.
- GPS devices recieve these signals . . .
- . . . and then use *trilateration*.

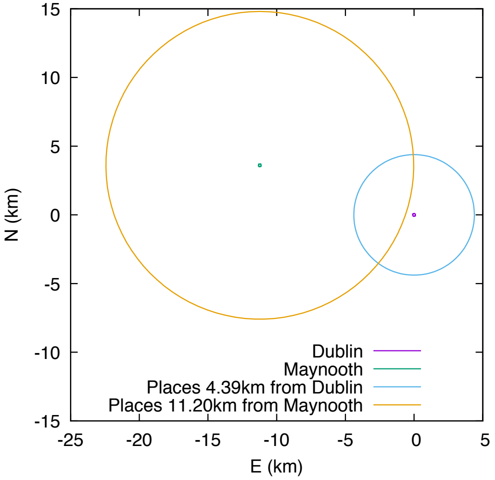
Trilateration



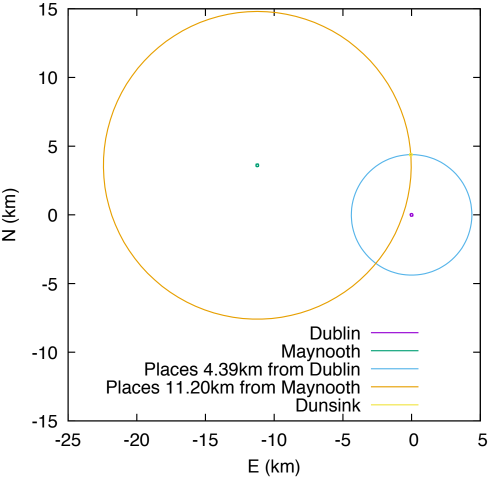
Trilateration



Trilateration



Trilateration



As equations, the distance between point 1 and point 2 is:

$$R_{1,2}^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2$$

So we need to solve equations:

$$R_{\text{Dunsink,Dublin}}^2 = (x_{\text{Dunsink}} - x_{\text{Dublin}})^2 + (y_{\text{Dunsink}} - y_{\text{Dublin}})^2$$

$$R_{\text{Dunsink,Maynooth}}^2 = (x_{\text{Dunsink}} - x_{\text{Maynooth}})^2 + (y_{\text{Dunsink}} - y_{\text{Maynooth}})^2$$

Where distances are in km and

- Dublin is at $(0, 0)$,
- Maynooth is at $(-11.229013, 3.608656)$
- $R_{\text{Dunsink,Dublin}} = 4.385669$ and
- $R_{\text{Dunsink,Maynooth}} = 11.199838$.

Suppose the GPS satellites all send a signal when our clock reads t. We don't know this time, but we do see when the signal arrives.

When satellite 1's signal reaches us, our clock reads t_1.
When satellite 2's signal reaches us, our clock reads t_2.
When satellite 3's signal reaches us, our clock reads t_3.
When satellite 4's signal reaches us, our clock reads t_4.
...

We know the signal moves at the speed of light, so $R_1 = c(t_1 - t)$. We also know (x_1, y_1, z_1) from the almanac. (Likewise for others)

Equations — this time we need 3D distance:

$$R_{1,2}^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2$$

So if we are at (x, y, z) and we can see four satellites:

$$R_1^2 = c^2(t - t_1)^2 = (x - x_1)^2 + (y - y_1)^2 + (z - z_1)^2$$

$$R_2^2 = c^2(t - t_2)^2 = (x - x_2)^2 + (y - y_2)^2 + (z - z_2)^2$$

$$R_3^2 = c^2(t - t_3)^2 = (x - x_3)^2 + (y - y_3)^2 + (z - z_3)^2$$

$$R_4^2 = c^2(t - t_4)^2 = (x - x_4)^2 + (y - y_4)^2 + (z - z_4)^2$$

Need 4 equations, since we have 4 unknowns.

If we solve, we get

- Our position (x, y, z).
- The time on our clock when the signal was sent t.
- This means GPS makes a really good clock!
- Usually good to 100 nannoseconds (0.0000001s).

To getting speed similar to position, but uses Doppler shift to each satellite.

- If you can see more satellites,
- . . . problem is usually *overspecified*,
- . . . so problem changes from solve to minimise.

Summary

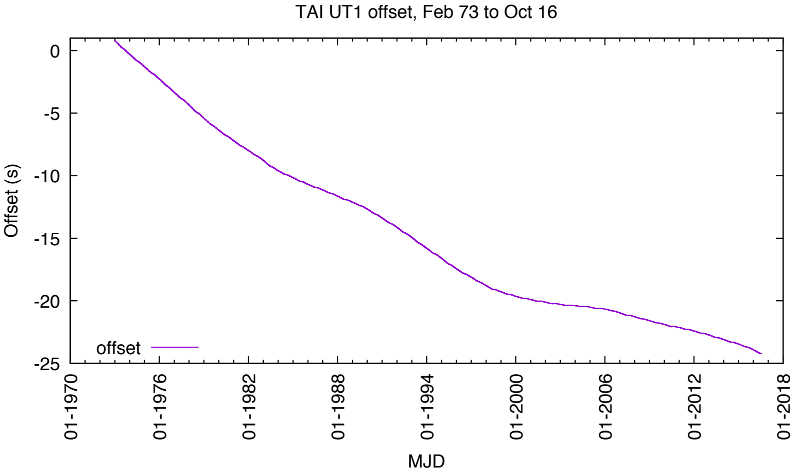
- Years and days relate to position/orientation of Earth and Sun.
- Comparing time often depends on seeing common event.
- Seconds are now defined atomically.
- GPS uses flying atomic clocks and trilatteration.

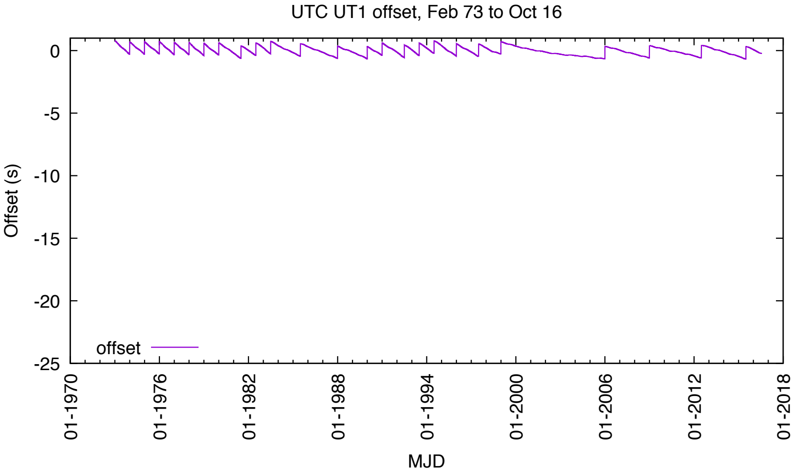
Used ET, based on Newcomb's measurements from 1750–1892.

Tidal forces (1.7ms/d/c) mean UT and SI seconds are different (by 2.5ms).

Coordinated Universal Time is a compromise. It ticks once per SI second, in sync with TAI.

If UTC is more than one second from UT1 then UTC is adjusted.


TAI UT1 offset, Feb 73 to Oct 16
Offset (s)
0
-5
-10
-15
-20
-25
offset
01-1970
01-1976
01-1982
01-1988
01-1994
01-2000
01-2006
01-2012
01-2018
MJD


UTC UT1 offset, Feb 73 to Oct 16
Offset (s)
0
-5
-10
-15
-20
-25
offset
01-1970
01-1976
01-1982
01-1988
01-1994
01-2000
01-2006
01-2012
01-2018
MJD

Time Dilation

Clocks that are moving seem to move more slowly. Scaled by

$$\sqrt{1 - \frac{v^2}{c^2}}$$

v is speed of clock and c is speed of light.
Clocks in a gravitational field also run more slowly

$$\sqrt{1 - \frac{2GM}{rc^2}}$$

G is gravitational constant, M is the mass of the object, r is distance from center and c is speed of light.

Doppler

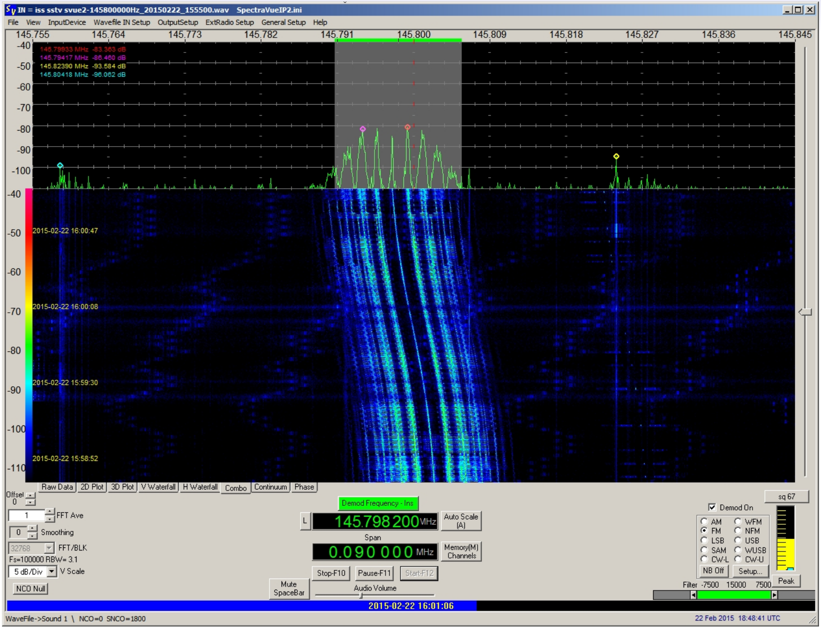
Waves pile up if you move towards source, or source moves towards you.

$$f = \frac{1 + \frac{v_r}{c}}{1 + \frac{v_s}{c}} f_0$$

If you aren't moving fast compared to the wave

$$f = \left(1 + \frac{\Delta v}{c}\right) f_0$$

Where Δv is the relative speed of source and receiver.
There is also a version that takes relativity into account!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145.79933 MHz -83.363 dB
145.79417 MHz -86.460 dB
145.82390 MHz -93.584 dB
145.80418 MHz -96.062 dB
2015-02-22 16:00:47
2015-02-22 16:00:08
2015-02-22 15:59:30
2015-02-22 15:58:52
Raw Data 2D Plot 3D Plot V Waterfall H Waterfall Combo Continuum Phase
Offset
FFT Ave
Smoothing
FFT/BLK
Fs=100000 RBW= 3.1
5 dB/Div V Scale
NCO Null
Demod Frequency - Ins
145.798 200 MHz
Auto Scale (A)
Span
0.090 000 MHz
Memory(M) Channels
Stop-F10 Pause-F11 Start-F12
Mute SpaceBar
Audio Volume
sq 67
Demod On
AM WFM FM NFM LSB USB SAM WUSB CW-L CW-U
NB Off Setup...
Filter -7500 15000 7500
Peak
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WaveFile->Sound 1 \ NCO=0 SNCO=1800
22 Feb 2015 18:48:41 UTC

NTP