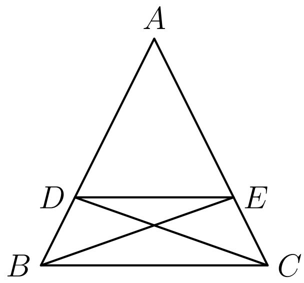
# MAU23302 Euclidean and Non-Euclidean Geometry
# Hilary Term 2021
# Some Sample Problems relating to Euclidean Geometry

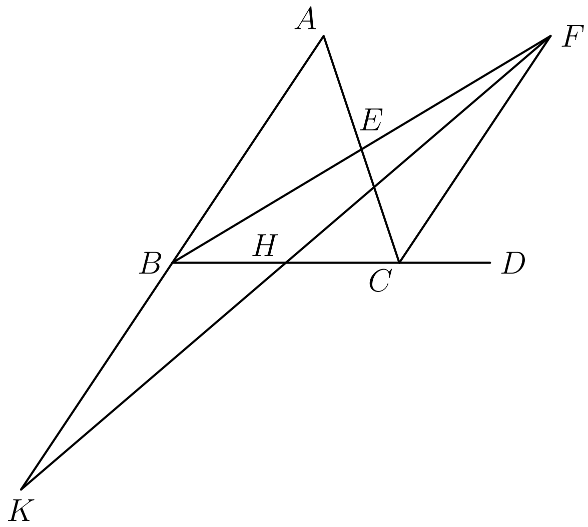
**Note.** Proofs answering the Euclidean geometry questions below should be based on propositions included in the first book of Euclid's *Elements of Geometry*. Whenever you apply propositions that succeed Proposition 15 in Book I of Euclid's *Elements*, you should cite the book and proposition number. Standard congruence rules may be cited as the SAS, SSS, ASA or SAA *Congruence Rule* as appropriate. Propositions 5 and 15 of Book I of Euclid's *Elements* may be cited as the *Isosceles Triangles Theorem* and the *Vertically-opposite Angles Theorem* respectively.

1. Let a geometric configuration in the Euclidean plane be as depicted below. In this configuration, we suppose that the straight line segments $AB$ and $AC$ are equal to one another in length, and also that the straight line segments $BD$ and $CE$ are equal to one another in length.




Making use of no proposition in Book I of Euclid's *Elements of Geometry* with the exception of the SAS Congruence Rule (Proposition 4), and *only applying Proposition 4 in cases where the triangles being compared represent distinct regions of the plane, and thus not attempting to apply Proposition 4 to compare two triangles where the second of those triangles is obtained from the first by merely relabelling the vertices of the first triangle*, prove that the angles $ABC$ and $ACB$ at the base of the isosceles triangle $ABC$ are equal to one another.

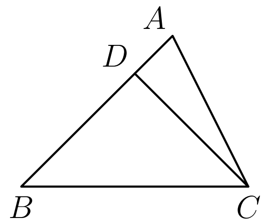
2. Let a geometric configuration in the Euclidean plane be as depicted below. In this configuration, we suppose that $E$ is the midpoint of the line segment $AC$ and also of the line segment $BF$, and that $H$ is the midpoint of the line segment $BC$, and also of the line segment $FK$.




Using only propositions from Euclid's *Elements of Geometry* that precede Proposition 16 in Book I, without making use of the Fifth Postulate, *and without assuming that the points $A$, $B$ and $K$ are collinear*, prove that the line segments $AB$ and $BK$ are equal in length.

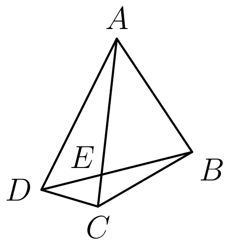
[Note: it would not be possible to prove that the points $A$, $B$ and $K$ are collinear without making use of either the Fifth Postulate, Proposition 29, or some proposition that follows Proposition 29 in Book I of Euclid's *Elements of Geometry.* In a typical matching configuration in hyperbolic geometry, no single geodesic would pass through all three of the points $A$, $B$ and $K$.]

3. Let $A$, $B$ and $C$ be the vertices of a triangle in the Euclidean plane in which the angle $ACB$ at the vertex $C$ is greater than the angle $ABC$ at the vertex $B$. Let a point $D$ be taken on the side $AB$ of this triangle so as to ensure that the angles $ABC$ and $BCD$ are equal to one another.




Without using any proposition in Book I of Euclid's *Elements of Geometry* that follows Proposition 15, with the exception of Proposition 20 (which ensures that the sum of the lengths of any two sides of a triangle is greater than the length of the remaining side), and assuming Proposition 20 to be valid and axiomatic, (and without essentially incorporating the proof of any proposition in Book I that follows Proposition 15 so as to avoid explicit citation of that proposition by reproving it,) prove that the side $AB$ of the triangle $ABC$ is in length greater than the side $AC$ of that triangle.

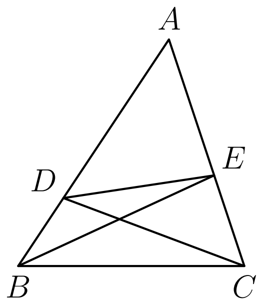
4. Let a geometric configuration in the Euclidean plane be as depicted below. In this configuration, we suppose that the point $C$ lies in the interior of the angle $DAB$, the line segments $AD$ and $AC$ are of equal length, and that the line segments $AC$ and $BD$ intersect at the point $E$.




Without using any proposition in Book I of Euclid's *Elements of Geometry* that follows Proposition 15, with the exception of Proposition 20 (which ensures that the sum of the lengths of any two sides of a triangle is greater than the length of the remaining side), and assuming Proposition 20 to be valid and axiomatic, (and without essentially incorporating the proof of any proposition in Book I that follows Proposition 15 so as to avoid explicit citation of that proposition by reproving it,) prove that the line segment $BD$ exceeds in length the line segment $BC$.

[Hint: consider the four triangles that meet at the vertex $E$ and apply the result stated in Proposition 20 to two of those triangles.]

5. Let a geometric configuration in the Euclidean plane be as depicted below. In this configuration, we suppose that the angle $ADE$ is less than the angle $ABC$.




Prove that the triangle $AEB$ is smaller in area than the triangle $ADC$.