Course MAU23203
Analysis in Several Real Variables.
Michaelmas Term 2022.
Assignment 1.

To be submitted on Blackboard on or before 11pm on Thursday 20th October, 2022.

Assignment submissions should be uploaded to Blackboard as a *single* file in PDF format. (Thus submissions should neither be constituted of multiple files, nor of photographic image files in a format such as JPEG or PNG.) It is recommended that a suitable scanner app be used.

Students are advised to retain a copy of their submission. It is anticipated that the assignment submissions would not be printed at any stage. Nor would they be annotated with hand-written comments: any feedback would most likely take the form of marks and comments forwarded separately to students to be interpreted in conjunction with copies of assignment submissions in their possession.

Students are reminded that they must comply with College policies with regard to plagiarism, which are published on the website located at the following URL:

`http://tcd-ie.libguides.com/plagiarism`

Please complete the cover sheet on the back of this page and attach it to the front of your completed assignment script, in particular signing the declaration with regard to plagiarism. Please make sure also that you include both name and student number on work handed in.

At an alternative to printing off the plagiarism declaration, it is acceptable to copy the wording of the plagiarism declaration on a sheet of paper and include the signed copy of the declaration, with name and student number, as a page included with the submission.

Solutions to problems should be expressed in appropriately concise and correct logical language. Attempted solutions that are incoherent, unclear, overlong or logically confused are unlikely to gain substantial credit.

Module MAU23203—Analysis in Several Real Variables.
Michaelmas Term 2022.
Assignment I.

Name (please print): . . . .

Student number: . . . .

Date submitted: . . . .

I have read and I understand the plagiarism provisions in the General Regulations of the University Calendar for the current year, found at

`http://www.tcd.ie/calendar`

I have also completed the Online Tutorial on avoiding plagiarism *Ready Steady Write*, located at

`http://tcd-ie.libguides.com/plagiarism/ready-steady-write`

Signed:

. . . .

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1. Let n be a positive integer, and let $\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \ldots$ be a bounded infinite sequence of points in $\mathbb{R}^n$. Suppose that the infinite sequence $\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \ldots$ does *not* converge to $\mathbf{0}$, where $\mathbf{0} = (0, 0, \ldots, 0)$. Prove that there exists a subsequence $\mathbf{x}_{k_1}, \mathbf{x}_{k_2}, \mathbf{x}_{k_3}, \ldots$ of the given infinite sequence which converges to some point $\mathbf{p}$ satisfying the condition $\mathbf{p} \neq \mathbf{0}$.

   [The required result may be proved by an application of the standard definition of convergence for infinite sequences in a Euclidean space, used in conjunction with a result, applicable to bounded sequences in Euclidean spaces, that is proved in the course notes.]

2. Throughout this question, let $f\colon \mathbb{R}^2 \to \mathbb{R}$ be the real-valued function on $\mathbb{R}^2$ defined such that

$$f(x, y) = \begin{cases} \dfrac{x^2}{y} & \text{whenever } y \neq 0; \\ 0 & \text{whenever } y = 0. \end{cases}$$

   (a) Is it the case that, for all $(u, v) \in \mathbb{R}^2$,

$$\lim_{t \to 0} f(tu, tv) = 0?$$

   [Your answer should be appropriately justified.]

   (b) Is it the case that that function f is continuous at $(0, 0)$? [Your answer should be appropriately justified.]