Course MAU23203
Analysis in Several Real Variables.
Michaelmas Term 2020.
Assignment 2.

To be submitted on Blackboard on or before 11pm on Friday 11th December, 2020.

Assignment submissions should be uploaded to Blackboard as a *single* file in PDF format. (Thus submissions should neither be constituted of multiple files, nor of photographic image files in a format such as JPEG or PNG.) It is recommended that a suitable scanner app be used.

Students are advised to retain a copy of their submission. It is anticipated that the assignment submissions would not be printed at any stage. Nor would they be annotated with hand-written comments: any feedback would most likely take the form of marks and comments forwarded separately to students to be interpreted in conjunction with copies of assignment submissions in their possession.

Students are reminded that they must comply with College policies with regard to plagiarism, which are published on the website located at the following URL:

`http://tcd-ie.libguides.com/plagiarism`

Please complete the cover sheet on the back of this page and attach it to the front of your completed assignment script, in particular signing the declaration with regard to plagiarism. Please make sure also that you include both name and student number on work handed in.

At an alternative to printing off the plagiarism declaration, it is acceptable to copy the wording of the plagiarism declaration on a sheet of paper and include the signed copy of the declaration, with name and student number, as a page included with the submission.

Module MAU22200—Analysis in Several Real Variables.
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Name (please print): ....

Student number: ....

Date submitted: ....

I have read and I understand the plagiarism provisions in the General Regulations of the University Calendar for the current year, found at

`http://www.tcd.ie/calendar`

I have also completed the Online Tutorial on avoiding plagiarism *Ready Steady Write*, located at

`http://tcd-ie.libguides.com/plagiarism/ready-steady-write`

Signed:

....

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Throughout this assignment, let $f\colon \mathbb{R}^2 \to \mathbb{R}$ be the real-valued function on the plane $\mathbb{R}^2$ defined so that $f(0,0) = 0$ and

$$f(x,y) = \frac{x^4 + y^4}{x^2 + y^2} \quad \text{whenever} \quad (x,y) \neq (0,0).$$

Moreover, for all points (p,q) of $\mathbb{R}^2$, let

$$f_x(p,q), \quad f_y(p,q), \quad f_{xx}(p,q), \quad f_{xy}(p,q), \quad f_{yx}(p,q), \quad f_{yy}(p,q)$$

denote the values of the functions

$$\frac{\partial f}{\partial x}, \quad \frac{\partial f}{\partial y}, \quad \frac{\partial^2 f}{\partial x^2}, \quad \frac{\partial^2 f}{\partial x\,\partial y}, \quad \frac{\partial^2 f}{\partial y\,\partial x}, \quad \frac{\partial^2 f}{\partial y^2}$$

respectively at the point (p,q).

1. Show that the second order partial derivatives of the function f exist at the point $(0,0)$, and determine the values of $f_x(0,0)$, $f_y(0,0)$, $f_{xx}(0,0)$, $f_{xy}(0,0)$, $f_{yx}(0,0)$ and $f_{yy}(0,0)$.

2. Write down expressions for the second order partial derivatives f_{xx}, f_{xy}, f_{yx} and f_{yy} at all points of the plane other than $(0,0)$.

   [You are simply asked to write down the relevant expressions. You are free to use software packages and or websites such as Wolfram Alpha to determine the values of the second order partial derivatives of the function at points other than $(0,0)$. **Do not show your working, if you determine these second order partial derivatives without the assistance of software. Do not submit your rough working. Simply make sure that your expressions are correct, and write down the correct expressions.**]

3. Are the second order partial derivatives of the function f continuous at the point $(0,0)$? [Briefly justify your answer.]

4. Let

$$\begin{aligned} g(x,y) &= f(x,y) - f(0,0) - x f_x(0,0) - y f_y(0,0) \\ &\quad - \tfrac{1}{2}x^2 f_{xx}(0,0) - \tfrac{1}{2}xy(f_{xy}(0,0) + f_{yx}(0,0)) - \tfrac{1}{2}y^2 f_{yy}(0,0). \end{aligned}$$

for all points (x,y) of $\mathbb{R}^2$. Determine an expression that specifies the value of the function g at all points (x,y) of the plane other than the origin.

5. Is it the case that the function g determined in the previous question has the property that

$$\lim_{(x,y)\to(0,0)} \frac{g(x,y)}{x^2+y^2} = 0?$$

[Briefly justify your answer.]