Course MAU23203: Michaelmas Term 2019.

Assignment 2.

To be handed in by Friday 22nd November, 2019.

Students are reminded that they must comply with College policies with regard to plagiarism, which are published on the website located at the following URL:

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Please complete the cover sheet on the back of this page and attach it to the front of your completed assignment script, in particular signing the declaration with regard to plagiarism. Please make sure also that you include both name and student number on work handed in.

Solutions to problems should be expressed in appropriately concise and correct logical language. Attempted solutions that are incoherent, unclear or logically confused will not gain substantial credit.

Module MAU23203—Analysis in Several Real Variables,
Michaelmas Term 2019.
Assignment II.

Name (please print): . . . .

Student number: . . . .

Date submitted: . . . .

I have read and I understand the plagiarism provisions in the General Regulations of the University Calendar for the current year, found at

`http://www.tcd.ie/calendar`

I have also completed the Online Tutorial on avoiding plagiarism *Ready Steady Write*, located at

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Signed:

. . . .

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Assignment 2.

1. This question concerns differentiation of a function of two real variables from first principles. Accordingly you may use freely any results concerning limits of functions stated and proved in Section 4 of the module notes, but you should not appeal to any lemma, proposition, theorem or corollary included in Section 9 of the notes.

   (a) Let p and q be real numbers. Then there exist real numbers r and s and a real-valued function $u\colon \mathbb{R}^2 \to \mathbb{R}$ of two real variables, where the constants r and s and the function u depend on and are uniquely determined by the values of p and q for which the following two properties both hold:

   $$x^3 - 3xy^2 = p^3 - 3pq^2 + r(x-p) - s(y-q) + u(x,y)$$

   for all real numbers x and y, and also

   $$\lim_{(x,y)\to(p,q)} \frac{1}{\sqrt{(x-p)^2 + (y-q)^2}} u(x,y) = 0.$$

   Determine the constants r and s and the function u, where those constants and that function are expressed in terms of the values of the constants p and q.

   (b) By making a straightforward substitution in the result proved in (a), or otherwise, determine the unique real-valued function $v\colon \mathbb{R}^2 \to \mathbb{R}$ for which the following two properties both hold:

   $$3x^2y - y^3 = 3p^2q - q^3 + s(x-p) + r(y-q) + v(x,y)$$

   for all real numbers x and y, and also

   $$\lim_{(x,y)\to(p,q)} \frac{1}{\sqrt{(x-p)^2 + (y-q)^2}} v(x,y) = 0.$$

   (c) Let $\varphi\colon \mathbb{R}^2 \to \mathbb{R}^2$ be the function from $\mathbb{R}^2$ to itself defined such that

   $$\varphi\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x^3 - 3xy^2 \\ 3x^2y - y^3 \end{pmatrix}$$

   Applying the results obtained in parts (a) and (b) of this question, determine, in terms of the values of p and q, the 2×2 matrix that represents the derivative of the function φ at a point (p,q) of $\mathbb{R}^2$.