

MA3486: Hemicontinuity Problem

Let $\Phi: \mathbb{R} \rightrightarrows \mathbb{R}$ be the correspondence defined such that

$$\Phi(x) = \begin{cases} \{y \in \mathbb{R} : xy^2 + (1 - x^2)y - x \geq 0\} & \text{if } x < 0; \\ \{y \in \mathbb{R} : y \geq 0\} & \text{if } x = 0; \\ \{y \in \mathbb{R} : y^2 - y - x - x^2 \leq 0\} & \text{if } 0 < x < 1; \\ \{y \in \mathbb{R} : 0 \leq xy \leq 2\} & \text{if } x \geq 1. \end{cases}$$

Answer the following, fully justifying your answers:—

- (i) Is $\Phi: \mathbb{R} \rightarrow \mathbb{R}$ is *upper hemicontinuous* at $x = 0$?
- (ii) Is $\Phi: \mathbb{R} \rightarrow \mathbb{R}$ is *lower hemicontinuous* at $x = 0$?
- (iii) Is $\Phi: \mathbb{R} \rightarrow \mathbb{R}$ is *upper hemicontinuous* at $x = 1$?
- (iv) Is $\Phi: \mathbb{R} \rightarrow \mathbb{R}$ is *lower hemicontinuous* at $x = 1$?