Course MA2321: Michaelmas Term 2018.

Assignment 2.

To be handed in by Monday 19th November, 2018.

Students are reminded that they must comply with College policies with regard to plagiarism, which are published on the website located at the following URL:

`http://tcd-ie.libguides.com/plagiarism`

Please complete the attached cover sheet and attach it to your assignment, in particular signing the declaration with regard to plagiarism. Please make sure also that you include both name and student number on work handed in.

Solutions to problems should be expressed in appropriately concise and correct logical language. Attempted solutions that are incoherent, unclear or logically confused will not gain substantial credit.

1. Throughout this question let $f \colon \mathbb{R} \to \mathbb{R}$ be the cosine function, where $f(x) = \cos x$, let b be real number satisfying $0 < b < \pi$, and, for each positive integer m, let P_m be the partition of $[0, b]$ given by $P_m = \{x_0, x_1, \ldots, x_m\}$ where $x_k = kb/m$ for $k = 0, 1, \ldots, m$.

   (a) Show that, for each positive integer m the upper sum $U(P_m, f)$ for the cosine function f satisfies $U(P_m, f) = F(b, b/m)$, where

$$F(b, h) = \frac{h(1 - \cos b - \cos h + \cos(b - h))}{2(1 - \cos h)}.$$

   [In answering part (a), you may find it helpful to make use of the following results:

$$\cos m\theta + i \sin m\theta = (\cos\theta + i \sin\theta)^m \quad \text{for all } m \in \mathbb{Z} \text{ where } i = \sqrt{-1};$$

$$\sum_{k=0}^{m-1} z^k = \frac{1 - z^m}{1 - z} \quad \text{for all } z \in \mathbb{C} \text{ satisfying } z \neq 1;$$

$$\frac{a + bi}{c + di} = \frac{(a + bi)(c - di)}{c^2 + d^2} \quad \text{for all } a, b, c, d \in \mathbb{R} \text{ with } (c, d) \neq (0, 0).]$$

   (b) Show that $\lim_{h \to 0} F(b, h) = \sin b$;

   [The following may be useful: $\lim_{h \to 0} \frac{2(1 - \cos h)}{h^2} = 1$; $\frac{d(\cos x)}{dx} = -\sin x$.]

   (c) Explain why $U(P_m, f) - L(P_m, f) \leq 2b/m$.

   (d) Without making use of the Fundamental Theorem of Calculus and its consequences, prove, on the basis of the results stated above, that the cosine function f is Riemann-integrable on the interval $[0, b]$, and that $\int_0^b \cos x = \sin b$.

2. Let f be a function that is 5 times differentiable on an open interval containing the closed interval $[a, b]$, where a and b are real numbers satisfying $a \leq b$. Suppose that

$$f(a) = f'(a) = f''(a) = f(b) = f'(b) = f''(b) = 0.$$

   Prove that there exists some real number s satisfying $a < s < b$ for which $f^{(5)}(s) = 0$.

Module MA2321—Analysis in Several Real Variables,
Michaelmas Term 2018.
Assignment II.

Name (please print): . . . .

Student number: . . . .

Date submitted: . . . .

I have read and I understand the plagiarism provisions in the General Regulations of the University Calendar for the current year, found at

`http://www.tcd.ie/calendar`

I have also completed the Online Tutorial on avoiding plagiarism *Ready Steady Write*, located at

`http://tcd-ie.libguides.com/plagiarism/ready-steady-write`

Signed:

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