Course MA2321: Michaelmas Term 2018.

Assignment 1.

To be handed in by Thursday 1st November, 2018.

Students are reminded that they must comply with College policies with regard to plagiarism, which are published on the website located at the following URL:

http://tcd-ie.libguides.com/plagiarism

Please complete the attached cover sheet and attach it to your assignment, in particular signing the declaration with regard to plagiarism. Please make sure also that you include both name and student number on work handed in.

Solutions to problems should be expressed in appropriately concise and correct logical language. Attempted solutions that are incoherent, unclear or logically confused will not gain substantial credit. 1. Let X be the subset of \mathbb{R}^3 defined such that

$$X = \{ (x, y, z) \in \mathbb{R}^3 : 1 \le x^2 + y^2 + z^2 < 4 \}.$$

Determine whether or not the set X is open in \mathbb{R}^3 . Determine also whether or not the set X is closed in \mathbb{R}^3 . [Justify your answers with appropriate logical reasoning or examples.]

2. Let Y be the subset of \mathbb{R}^2 defined such that

$$Y = \{(x, y) \in \mathbb{R}^2 : \text{there exists } n \in \mathbb{Z} \text{ such that } (x - n)^2 + y^2 < 1\}.$$

(In other words, a point (x, y) of \mathbb{R}^2 belongs to Y if and only if some integer n, depending on the values of x and y, can be found for which $(x - n)^2 + y^2 < 1$.) Determine whether or not the set Y is open in \mathbb{R}^2 . Determine also whether or not the set Y is closed in \mathbb{R}^2 . [Justify your answers with appropriate logical reasoning or examples.]

3. Let Z be the subset of \mathbb{R}^2 defined such that

$$Z = \{(x, y) \in \mathbb{R}^2 : \text{there exists } n \in \mathbb{Z} \text{ such that } (x - n)^2 + y^2 \le 1\}.$$

Determine whether or not the set Z is open in \mathbb{R}^2 . Determine also whether or not the set Z is closed in \mathbb{R}^2 . [Justify your answers with appropriate logical reasoning or examples.]

4. Let $f: \mathbb{R}^2 \to \mathbb{R}$ be the function of two real-variables defined such that

$$f(x,y) = \begin{cases} \frac{4x(x^2 + y^2)}{4x^2 + (x^2 + y^2)^2} & \text{if } (x,y) \neq (0,0); \\ 0 & \text{if } (x,y) = (0,0). \end{cases}$$

(a) Determine the values of the function f at all points lying on the circle of radius 1 centred at the point (1,0).

(b) Prove that $\lim_{t\to 0} f(tu, tv) = 0$ for all points (u, v) of \mathbb{R}^2 .

(c) Determine whether or not the function $f: \mathbb{R}^2 \to \mathbb{R}$ is continuous at (0,0). [Justify your answer with appropriate logical reasoning or examples.]

Module MA2321—Analysis in Several Real Variables, Michaelmas Term 2018. Assignment I.

Name (please print):

Student number:

Date submitted:

I have read and I understand the plagiarism provisions in the General Regulations of the University Calendar for the current year, found at

http://www.tcd.ie/calendar

I have also completed the Online Tutorial on avoiding plagiarism *Ready* Steady Write, located at

http://tcd-ie.libguides.com/plagiarism/ready-steady-write
Signed: