Course MA2321: Michaelmas Term 2017.

Assignment II.

To be handed in by Friday 26th January, 2018.

Students are reminded that they must comply with College policies with regard to plagiarism, which are published on the website located at the following URL:

`http://tcd-ie.libguides.com/plagiarism`

Please complete and sign the cover sheet below, and attach it to your assignment, in particular signing the declaration with regard to plagiarism. Please make sure also that both name and student number are included on any work handed in.

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I have also completed the Online Tutorial on avoiding plagiarism *Ready Steady Write*, located at

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Signed:

....

Module MA2321—Analysis in Several Real Variables.
Michaelmas Term 2017.
Assignment II

1. Let $\cosh y = \frac{1}{2}(e^y + e^{-y})$ and $\sinh y = \frac{1}{2}(e^y - e^{-y})$ for all real numbers y, and let $\varphi\colon \mathbb{R}^2 \to \mathbb{R}^2$ be the mapping from $\mathbb{R}^2$ to $\mathbb{R}^2$ defined such that

$$\varphi(x, y) = (\cos x\ \cosh y, -\sin x \sinh y)$$

for all real numbers x and y. Let $(p, q) \in \mathbb{R}^2$. Determine the derivative $(D\varphi)_{p.q}$ of the mapping φ at (p, q), and determine the values of real numbers r and θ with the property that

$$(D\varphi)_{p,q} = \begin{pmatrix} r\cos\theta & -r\sin\theta \\ r\sin\theta & r\cos\theta \end{pmatrix},$$

expressing r and $\cos\theta$ and $\sin\theta$ in terms of p and q.

2. For each positive real number k, let f_k be the function from $\mathbb{R}^2$ to $\mathbb{R}$ defined such that $f_k(0, 0) = 0$ and

$$f_k(x, y) = \frac{x^4 + y^4}{(x^2 + y^2)^k}$$

for all points (x, y) of $\mathbb{R}^2$ distinct from $(0, 0)$. Determine the values of the positive real number k for which the corresponding function f_k is continuous at $(0, 0)$. Determine also the values of the positive integer k for which the corresponding function f_k is differentiable at $(0, 0)$.