Course MA2321: Michaelmas Term 2017.

Assignment 1.

To be handed in by Thursday 23rd November, 2017.

Students are reminded that they must comply with College policies with regard to plagiarism, which are published on the website located at the following URL:

`http://tcd-ie.libguides.com/plagiarism`

Please complete the attached cover sheet and attach it to your assignment, in particular signing the declaration with regard to plagiarism. Please make sure also that you include both name and student number on work handed in.

1. Let a, b and c be fixed real numbers, and let

$$X = \{(x, y, z) \in \mathbb{R}^3 : x < a \text{ and } y < b \text{ and } z < c\}.$$

(a) Let $(u, v, w) \in X$. What is the largest value of δ for which the open ball of radius δ about the point (u, v, w) is contained in the set X? [Express δ in terms of a, b, c, u, v and w.]

(b) Is the set X closed in $\mathbb{R}^3$? [Justify your answer.]

2. Let $g\colon \mathbb{R} \to \mathbb{R}$ and $h\colon \mathbb{R} \to \mathbb{R}$ be continuous real-valued functions on $\mathbb{R}$. Prove that

$$\{(x, y) \in \mathbb{R}^2 : g(x) < y < h(x)\}$$

is an open set in $\mathbb{R}^2$. [Hint: consider ways in which you could apply Proposition 4.18 of the lecture notes for module MA2321 in Michaelmas Term 2017.]

3. Let $f\colon \mathbb{R}^2 \to \mathbb{R}$ be the real-valued function on $\mathbb{R}^2$, defined so that

$$f(x, y) = \begin{cases} \dfrac{2xy^2}{(x^2 + y^2)^2} & \text{if } (x, y) \neq (0, 0); \\ 0 & \text{if } (x, y) = (0, 0). \end{cases}$$

Standard theorems of analysis in one and several variables ensure that the function f is continuous at all points (x, y) for which $(x, y) \neq (0, 0)$. (See in particular Proposition 4.5 in the lecture notes for module

MA2321 Notes in Michaelmas Term 2017.) Also the partial derivatives $\dfrac{\partial f(x,y)}{\partial x}$ and $\dfrac{\partial f(x,y)}{\partial y}$ are defined for all $(x,y) \in \mathbb{R}^2$. Indeed if $(x,y) \neq (0,0)$ then

$$\frac{\partial f(x,y)}{\partial x} = \frac{2y^2(y^2 - 3x^2)}{(x^2+y^2)^3}, \qquad \frac{\partial f(x,y)}{\partial y} = \frac{4xy(x^2-y^2)}{(x^2+y^2)^3},$$

and if $(x,y) = 0$ then

$$\left.\frac{\partial f(x,y)}{\partial x}\right|_{(0,0)} = 0, \qquad \left.\frac{\partial f(x,y)}{\partial y}\right|_{(0,0)} = 0,$$

because $f(x,0) = 0$ for all real numbers x and $f(0,y) = 0$ for all real numbers y.

(a) Determine, for each real number x, the value of $\lim\limits_{y \to 0} f(x,y)$.

(b) Let $g\colon \mathbb{R} \to \mathbb{R}$ be the function on $\mathbb{R}$ determined such that $g(y) = \int\limits_{x=0}^{1} f(x,y)\,dx$ for all real numbers y. Determine the value of $g(y)$ for all real numbers y, and determine the value of $\lim\limits_{y \to 0} g(y)$.

(c) Is it the case that

$$\lim_{y \to 0} \int_{x=0}^{1} f(x,y)\,dx = \int_{x=0}^{1} \left(\lim_{y \to 0} f(x,y) \right)\,dx?$$

(d) For each real number b, determine the maximum value of $f(x,y)$ on the line $y = b$, and determine the value of x for which $f(x,b)$ attains its maximum value.

(e) Is the function f continuous at $(0,0)$? [Fully justify your answer.]

Module MA2321—Analysis in Several Real Variables, Michaelmas Term 2017.
Assignment I.

Name (please print): . . . .

Student number: . . . .

Date submitted: . . . .

I have read and I understand the plagiarism provisions in the General Regulations of the University Calendar for the current year, found at

`http://www.tcd.ie/calendar`

I have also completed the Online Tutorial on avoiding plagiarism *Ready Steady Write*, located at

`http://tcd-ie.libguides.com/plagiarism/ready-steady-write`

Signed:

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