MA2224, Hilary Term Examination 2019
Syllabus of Examinable Material

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General Remarks

The lists below specify the subsections of the course material that contain particular results (lemmas, propositions, theorems, corollaries) that candidates are expected to know and produce at the examination. In some cases the result is specified as "statement only": in such cases candidates will not be asked to give proofs on the examination paper. Where proofs are examined, it is not expected that candidates reproduce a verbatim reproduction of a proof in the printed notes. If asked to prove a result, any valid proof (within the framework of the module) is acceptable. Indeed candidates, in preparing for the examination, may decide to focus on the key steps within a particular proof, with confidence that they can complete the proof on the basis of their general competence, combined with an overview of the basic strategy and significant steps of the proof. If another numbered result is needed as a prerequisite for a given proof, it can normally be presumed that the prerequisite result can be stated without proof. If a proof were expected of a prerequisite result, then the examination paper would have been drafted to explicitly require that proof in a previous part of the question.

Section 1

Subsections 1.1 to 1.6 are not examinable.

The following material from subsection 1.7 is examinable:

> the definition of the *upper limit* of a bounded sequence of real numbers
> the definition of the *lower limit* of a bounded sequence of real numbers
> the statement and proof of Proposition 1.12

The following material from subsection 1.8 is examinable:

the statement and proof of Proposition 1.13
the statement and proof of Corollary 1.14
the statement and proof of Corollary 1.15

Subsections 1.9 to 1.12 are not examinable.
The following material from subsection 1.13 is examinable:

the definition of the *extended real number system*
the definitions of *addition* and *multiplication*
for sums and products involving $+\infty$ and $-\infty$

Section 2

The following material from subsection 2.1 is examinable:

the definition of an n-dimensional *block*
the definition of the content of an n-dimensional *block*
the result (but not the proof) of Proposition 2.2
the result (but not the proof) of Proposition 2.3
the result of Corollary 2.4
the result (but not the proof) of Lemma 2.5
the statement and proof of Proposition 2.6

The following material from subsection 2.2 is examinable:

the definition of *Lebesgue outer measure*
the statement and proof of Lemma 2.7
the statement (but not the proof) of Lemma 2.8
the statement and proof of Proposition 2.9
the statement (but not the proof) of Proposition 2.10

The following material from subsection 2.3 is examinable:

the definition of *outer measure*
the definition of λ-measurable sets for an outer measure λ
the statement and proof of Proposition 2.11
the statement and proof of Lemma 2.12
the statement and proof of Proposition 2.13
the statement and proof of Corollary 2.14
the statement and proof of Proposition 2.15

The following material from subsection 2.4 is examinable:

the definition of σ-*algebra*
the statement and proof of Lemma 2.16
the definition of a *measure* on a σ-algebra
the statement (but not the proof) of Proposition 2.17
the statement and proof of Lemma 2.18

The following material from subsection 2.5 is examinable:

the definition of *Lebesgue-measurable* subsets of $\mathbb{R}^n$
the statement (but not the proof) of Proposition 2.19
the statement and proof of Proposition 2.20
the statement and proof of Corollary 2.21
the definition of *Borel sets* in $\mathbb{R}^n$
the definition of a *Borel measure*
the statement (but not the proof) of Corollary 2.22

The following material from subsection 2.6 is examinable:

the statement and proof of Lemma 2.23
the statement and proof of Lemma 2.24

Subsection 2.7 is not examinable.

Section 3

The following material from subsection 3.1 is examinable:

the definition of *measurability* of a function on a measure space
the statement and proof of Proposition 3.1
the statement and proof of Corollary 3.2
the statement (but not the proof) of Lemma 3.3
the statement and proof of Proposition 3.4
the statement and proof of Proposition 3.5
the statement and proof of Proposition 3.6
the statement (but not the proof) of Proposition 3.8
the statement and proof of Lemma 3.9
the statement and proof of Proposition 3.10
the statement (but not the proof) of Corollary 3.11

The following material from subsection 3.2 is examinable:

the definition of the *characteristic function* of a subset
the definition of *integrable simple function*
the definition of the *integral of an integrable simple function*
the statement (but not the proof) of Corollary 3.15
the statement (but not the proof) of Proposition 3.16
the statement (but not the proof) of Corollary 3.17
the statement (but not the proof) of Corollary 3.19

The following material from subsection 3.3 is examinable:

the statement (but not the proof) of Proposition 3.20
the definition of the *integral* of a non-negative measurable function
the statement (but not the proof) of Lemma 3.21

The following material from subsection 3.4 is examinable:

the statement and proof of Theorem 3.22
the statement (but not the proof) of Proposition 3.23
the statement and proof of Proposition 3.24

The following material from subsection 3.5 is examinable:

the statement and proof of Lemma 3.25

The following material from subsection 3.6 is examinable:

the definition of *integrable* function on a measure space
the definition of the *integral* of an integrable function
the statement and proof of Lemma 3.26
the statement (but not the proof) of Lemma 3.27

The following material from subsection 3.7 is examinable:

the statement and proof of Theorem 3.28

Subsection 3.8 is not examinable.
Subsection 3.9 is not examinable.