Course 214, Annual Exam 2007, Question 1(c) Worked Solution

The set $\{z \in \mathbb{C} : |z - 2| < 1$ and $|z - 3| < 1\}$

This set is open. It is the intersection of the open disk of radius 1 centered on 2 and the open disk of radius 1 centered on 3. Open disks are open sets (1(b)), and the intersection of a finite number of open sets is an open set.

This set is not closed. For example, the value 2 is in the complement of the set, but every open disk of positive radius about 2 intersects the set. It follows that the complement of the set is not open, and therefore the set itself is not closed.

The set $\{z \in \mathbb{C} : |z - 2| < 1$ and $|z - 3| \leq 1\}$

This set is not open. Indeed the value 2 belongs to the set, but no open disk of positive radius about 2 is contained in the set.

This set is not closed. For example, the value 3 is in the complement of the set, but every open disk of positive radius about 3 intersects the set. It follows that the complement of the set is not open, and therefore the set itself is not closed.

The set $\{z \in \mathbb{C} : |z - 2| \leq 1$ and $|z - 3| \leq 1\}$

This set is not open. Indeed the value 2 belongs to the set, but no open disk of positive radius about 2 is contained in the set.

This set is closed. The set $\{z \in \mathbb{C} : |z - 2| > 1\}$ is open, by a standard result, and therefore its complement is closed. Thus $\{z \in \mathbb{C} : |z - 2| \leq 1\}$ is closed. Similarly $\{z \in \mathbb{C} : |z - 3| \leq 1\}$ is closed. The given set is the intersection of these two closed sets, and the intersection of any number of closed sets is closed. Therefore the given set is closed.