Course 214: Hilary Term 2007.

Assignment II.

To be handed in by Monday 19th February, 2007.
Please include both name and student number on any work handed in.

1. Evaluate
$$\int_{-\infty}^{\infty} \frac{e^{isx}\,dx}{(x^2+1)(x^2+4)}$$
for all positive real numbers s. [Hint: apply Cauchy's Residue Theorem to the path integral taken around a closed path (or contour) that traverses the real axis from $-R$ to R, where R is a large positive real number, and then returns from R to $-R$ in the anticlockwise direction around a semicircle of radius R about zero in the upper half plane.]

2. Evaluate
$$\int_0^{+\infty} \frac{x^\alpha\,dx}{x(x^2+1)(x^2+4)}$$
for all real numbers α satisfying $0 < \alpha < 1$. [Hint: let R be a large positive real number, let e be a small positive real number, and apply Cauchy's Residue Theorem to the path integral of
$$\frac{z^\alpha}{z(z^2+1)(z^2+4)}$$
taken round a contour that traverses a straight line from $-R+ie$ to $(1+i)e$, then traverses part of a circle of radius $\sqrt{2}e$ centered on zero in a clockwise direction from $(1+i)e$ to $(1-i)e$, then traverses a straight line from $(1-i)e$ to $-R-ie$, and finally traverses part of a circle of radius $\sqrt{R^2+e^2}$ centered on zero in an anti-clockwise direction from $-R-ie$ to $-R+ie$. Then let $R \to +\infty$ and $e \to 0$. Here $z^\alpha = \exp(\alpha \log z)$, where $\log z$ denotes the principal branch of the logarithm function, defined throughout $\mathbb{C} \setminus \{t \in \mathbb{R} : t \leq 0\}$.]