Relations for conjugating spinors

Donie O'Brien
Email: donie@maths.tcd.ie

The hermitian congugates of spinors $u = u(\vec{p}, \lambda)$ are as follows

$$\begin{aligned} u^\dagger &= \bar{u}\,\gamma^0 \\ \bar{u}^\dagger &= \left(\gamma^0\right)^\dagger u = \gamma^0\, u \\ \gamma_\mu^\dagger &= \gamma_0\,\gamma_\mu\,\gamma_0 \\ \gamma^0 &= \gamma_0 \\ (\gamma_0)^2 &= \gamma^0\,\gamma_0 = I_{4X4} \\ (\gamma_0)^\dagger &= \gamma_0 \end{aligned}$$

The completness relations for the sum of spinors are

$$\sum_\lambda u(\vec{p}, \lambda)\,\bar{u}(\vec{p}, \lambda) = (\,\gamma \cdot p + m_p\,)$$

$$\sum_{\lambda'} v(\vec{k}, \lambda')\,\bar{v}(\vec{k}, \lambda') = (\,\gamma \cdot k - m_k\,)$$

where u are the spinors for particles, and v are the spinors for antiparticles. The spinors satisfy the Dirac equation as follows:

$$\begin{aligned} (\,\gamma \cdot p - m\,)\,u(p) &= 0 \\ (\,\gamma \cdot p + m\,)\,v(p) &= 0 \end{aligned}$$

and conjugating these equations we obtain

$$\begin{aligned} \bar{u}(p)\,(\,\gamma \cdot p - m\,) &= 0 \\ \bar{v}(p)\,(\,\gamma \cdot p + m\,) &= 0 \end{aligned}$$

The Dirac equation is

$$(\,i\gamma^\mu \partial_\mu - m\,)\,\psi(x) = 0$$

where the plane wave solutions are

$$\psi(x) = u(p)e^{-ip\cdot x}$$

for particles and

$$\psi(x) = v(p)e^{+ip\cdot x}$$

for antiparticles.