Spin observables for polarizing antiprotons

Donie O'Brien

Trinity College Dublin, Ireland

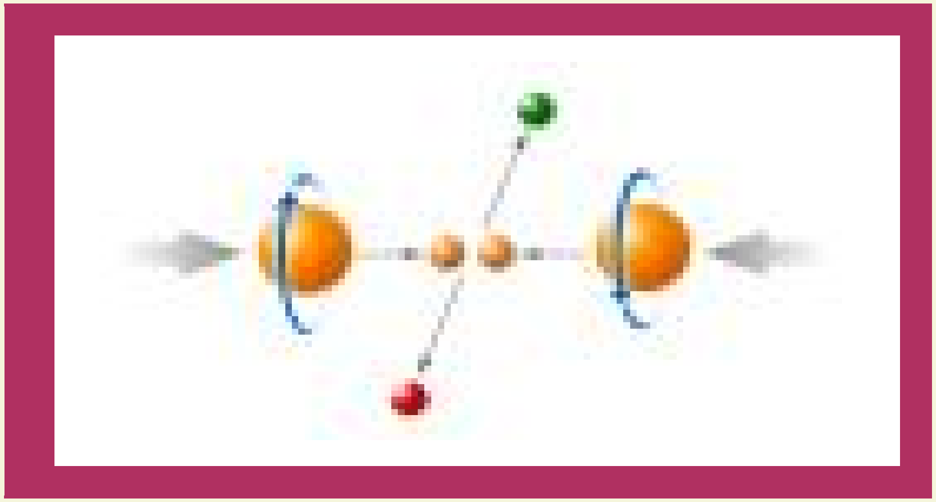
Email: donie@maths.tcd.ie

3 - October - 2006

Introduction

- The $\mathcal{PAX}$ project at GSI Darmstadt plans to polarize an antiproton beam by repeated interaction with a hydrogen target in a storage ring.
- Many of the beam particles are required to remain within the ring after interaction with the target so small scattering angles are important. Hence we concentrate on low momentum transfer (small t).
- Electromagnetic effects dominate the hadronic effects in this low t region of interest. Thus we calculate all Electromagnetic Helicity amplitudes and Spin Observables for elastic $\bar{p}\,p$ and $\bar{p}\,e$ scattering, to first order in QED.
- A beam of polarized electrons with energy sufficient to provide scattering of antiprotons beyond ring acceptance may polarize an antiproton beam by spin filtering.
- The spin observables are then used to estimate the rate of buildup of polarization of an antiproton beam.

1 The Electromagnetic Helicity Amplitudes and Spin Observables



1.1 The Generic Calculation

The generic equation for polarization effects in elastic spin 1/2 - spin 1/2 scattering to first order in QED is

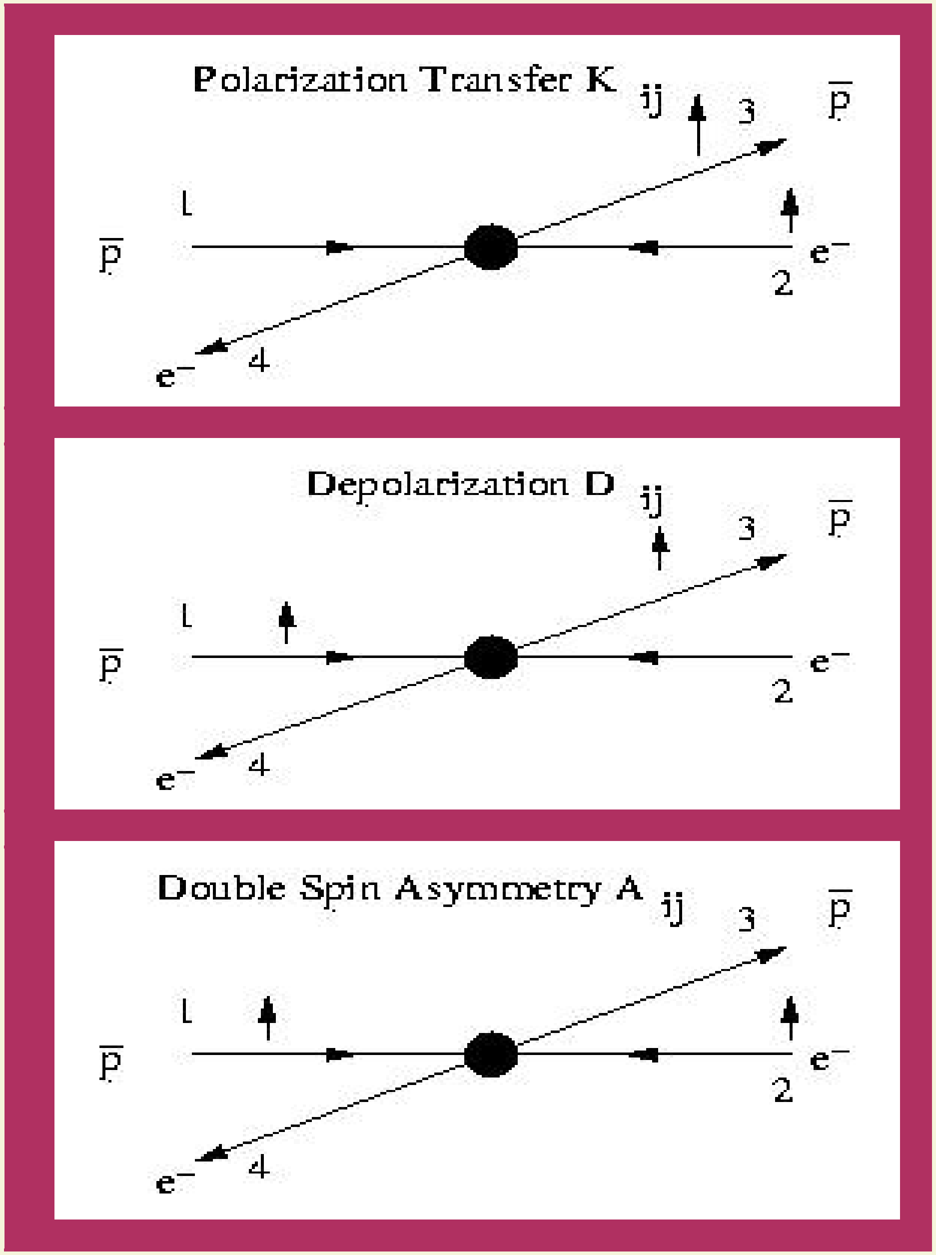
$$16\left(\frac{q}{e}\right)^4 |\mathcal{M}|^2 =$$

$$\text{Tr}\left[\left(\not{p}_4 + m\right)(1+\gamma_5\not{S}_4)(g_M\gamma^\nu + f r^\nu)\left(\not{p}_2 + m\right)(1+\gamma_5\not{S}_2)(g_M\gamma^\mu + f r^\mu)\right]\times$$

$$\text{Tr}\left[\left(\not{p}_1 + M\right)(1+\gamma_5\not{S}_1)(G_M\gamma_\mu + F R_\mu)\left(\not{p}_3 + M\right)(1+\gamma_5\not{S}_3)(G_M\gamma_\nu + F R_\nu)\right]$$

where the electromagnetic form factors $G_M = F_1 + F_2$, $g_M = f_1 + f_2$, $F = -F_2/2M$ and $f = -f_2/2m$; also $R^\mu = p_1^\mu + p_3^\mu$, $r^\mu = p_2^\mu + p_4^\mu$. This generic equation can thus be used to calculate all helicity amplitudes and spin observables etc. by substituting specific values for the spin (S_i) and momenta (p_i) vectors. The result has been obtained for this equation with the traces computed and contracted, using *Mathematica*.

Centre-of-Mass Momenta vectors	
$p_1 = (E_1,\ 0,\ 0,\ k)$	$p_3 = (E_1,\ k\sin\theta,\ 0,\ k\cos\theta)$
$p_2 = (E_2,\ 0,\ 0,\ -k)$	$p_4 = (E_2,\ -k\sin\theta,\ 0,\ -k\cos\theta)$
Centre-of-Mass Normal spin vectors	
$S_1^N = (0,\ 0,\ 1,\ 0)$	$S_3^N = (0,\ 0,\ 1,\ 0)$
$S_2^N = (0,\ 0,\ 1,\ 0)$	$S_4^N = (0,\ 0,\ 1,\ 0)$
Centre-of-Mass Transverse spin vectors	
$S_1^T = (0,\ 1,\ 0,\ 0)$	$S_3^T = (0,\ \cos\theta,\ 0,\ -\sin\theta)$
$S_2^T = (0,\ 1,\ 0,\ 0)$	$S_4^T = (0,\ -\cos\theta,\ 0,\ \sin\theta)$
Centre-of-Mass Longitudinal spin vectors	
$S_1^L = \frac{1}{M}(k,\ 0,\ 0,\ E_1)$	$S_3^L = \frac{1}{M}(k,\ E_1\sin\theta,\ 0,\ E_1\cos\theta)$
$S_2^L = \frac{1}{m}(k,\ 0,\ 0,\ -E_2)$	$S_4^L = \frac{1}{m}(k,\ -E_2\sin\theta,\ 0,\ -E_2\cos\theta)$


Polarization Transfer K
ij
3
p̄
L
p̄
e⁻
2
e⁻
4
Depolarization D
ij
3
p̄
L
p̄
e⁻
2
e⁻
4
Double Spin Asymmetry A
ij
3
p̄
L
p̄
e⁻
2
e⁻
4

1.2 Helicity Amplitudes

The notation of the helicily amplitudes $\mathcal{M}(\,A', B'\,;\,A, B\,)$ is $\mathcal{M}(\,\pm, \pm\,;\,\pm, \pm\,)$ where the arguments are $+$ if the spin vector is as S_i^L above (polarized along the direction of motion) and $-$ if the spin vector is minus S_i^L above (polarized opposite to the direction of motion). After using T and P invariance there are 6 independent helicity amplitudes for the scattering of two non-identical spin $1/2$ particles.

$$\begin{aligned} \phi_1 &\equiv \mathcal{M}(+,+\,;+,+) & \phi_2 &\equiv \mathcal{M}(+,+\,;-,-) \\ \phi_3 &\equiv \mathcal{M}(+,-\,;+,-) & \phi_4 &\equiv \mathcal{M}(+,-\,;-,+) \\ \phi_5 &\equiv \mathcal{M}(+,+\,;+,-) & \phi_6 &\equiv \mathcal{M}(+,+\,;-,+) \end{aligned}$$

Note for $p\,p$, $\bar{p}\,p$ and $\bar{p}\,\bar{p}$ scattering $\phi_6 = -\phi_5$, so there are only 5 independent helicity amplitudes.

1.3 First order QED results

$$\frac{\phi_1}{\alpha} = \frac{s-m^2-M^2}{t}\left(1+\frac{t}{4\,k^2}\right) f_1\,F_1 - f_1\,F_1 - f_2\,F_1 - f_1\,F_2 - \frac{1}{2} f_2\,F_2 \left(1-\frac{t}{4\,k^2}\right)$$

$$\frac{\phi_2}{\alpha} = \frac{1}{2}\left(\frac{m}{k} f_1 - \frac{k}{m} f_2\right)\left(\frac{M}{k} F_1 - \frac{k}{M} F_2\right) + \frac{s-m^2-M^2-2\,k^2}{4\,m\,M}\left(1+\frac{t}{4\,k^2}\right) f_2\,F_2$$

$$\frac{\phi_3}{\alpha} = \left[\frac{s-m^2-M^2}{t} f_1\,F_1 + \frac{f_2\,F_2}{2}\right]\left(1+\frac{t}{4\,k^2}\right)$$

$$\phi_4 = -\phi_2$$

$$\frac{\phi_5}{\alpha} = \sqrt{\frac{s}{-t}\,(4\,k^2+t)}\left[\frac{f_1\,F_1\,m}{4\,k^2}\left(1-\frac{m^2-M^2}{s}\right) - \frac{f_2\,F_1}{2\,m} + \frac{t\,f_2\,F_2}{16\,m\,k^2}\left(1+\frac{m^2-M^2}{s}\right)\right]$$

$$\frac{\phi_6}{\alpha} = \sqrt{\frac{s}{-t}\,(4\,k^2+t)}\left[\frac{f_1\,F_1\,M}{4\,k^2}\left(\frac{M^2-m^2}{s}-1\right) + \frac{f_1\,F_2}{2\,M} - \frac{t\,f_2\,F_2}{16\,M\,k^2}\left(1+\frac{M^2-m^2}{s}\right)\right]$$

1.4 Spin Observables

- All the electromagnetic spin observables of a reaction (polarization transfer K_{ij}, depolarization $(1-D_{ij})$ and asymmetries A_{ij} where $i, j, k \in \{\mathrm{X}, \mathrm{Y}, \mathrm{Z}\}$) can now be obtained by direct computation. See D.O'B. and N. H. Buttimore hep-ph/0609233 for complete results.

- For electromagnetic interactions to first order the double spin asymmetries equal the polarization transfer observables ($A_{ij} = K_{ij}$) and all the single and triple spin asymmetries are zero ($A_i = A_{ijk} = 0$).

- Spin filtering requires evaluation of the angular integration of the product of the observables $A_{ii} = K_{ii}$ and $(1-D_{ii})$ with $d\,\sigma/d\,\Omega$. Azimuthal averaging indicates that the observables with single X (i.e. K_{XZ}, K_{ZX}, D_{XZ} and D_{ZX}) do not contribute to spin filtering. The quantities $(K_{\mathrm{XX}} + K_{\mathrm{YY}})/2$, $(D_{\mathrm{XX}} + D_{\mathrm{YY}})/2$, K_{ZZ} and D_{ZZ} play the important role, we now present results for these.

2 Antiproton-proton scattering

To look at the case of antiproton-proton scattering set the form factors and masses of each particle equal ($f_1 \to F_1$, $f_2 \to F_2$ and $m \to M$) in the generic equation. We obtain the results to leading order in small t:

$$\frac{K_{\mathrm{XX}} + K_{\mathrm{YY}}}{2} \frac{d\sigma}{d\Omega} \approx \frac{\alpha^2 M^2 \mu^2}{s\, t}$$

$$\frac{(1 - D_{\mathrm{XX}}) + (1 - D_{\mathrm{YY}})}{2} \frac{d\sigma}{d\Omega} \approx \frac{-\alpha^2 \left(k^2 + M^2\right)}{k^2 M^2 s\, t} \left[M^2 - 2 k^2 (\mu - 1)\right]^2$$

$$K_{\mathrm{ZZ}} \frac{d\sigma}{d\Omega} \approx \frac{-2 \alpha^2 \mu^2}{s\, t} \left(2 k^2 + M^2\right)$$

$$(1 - D_{\mathrm{ZZ}}) \frac{d\sigma}{d\Omega} \approx \frac{-2 \alpha^2 \left(k^2 + M^2\right)}{k^2 M^2 s\, t} \left[M^2 - 2 k^2 (\mu - 1)\right]^2$$

2.1 Antiproton-electron scattering

To look at the case of antiproton-electron scattering set the form factors of the second particle to be structureless ($f_1 \to 1$ and $f_2 \to 0$) in the generic equation. We obtain the results to leading order in small t:

$$\frac{K_{\rm XX} + K_{\rm YY}}{2}\,\frac{d\,\sigma}{d\,\Omega} \approx \frac{\alpha^2\; m\; M\;\; \mu}{s\; t}$$

$$\frac{(1 - D_{\rm XX}) + (1 - D_{\rm YY})}{2}\,\frac{d\,\sigma}{d\,\Omega} \approx \frac{-m^2\,\alpha^2\,\left(s - m^2 + M^{\,2}\right)^2}{4\; k^2\; s^2\; t}$$

$$K_{\rm ZZ}\,\frac{d\,\sigma}{d\,\Omega} \approx \frac{-\,\alpha^2\;\mu}{s\; t}\left(s - m^2 - M^{\,2}\right)$$

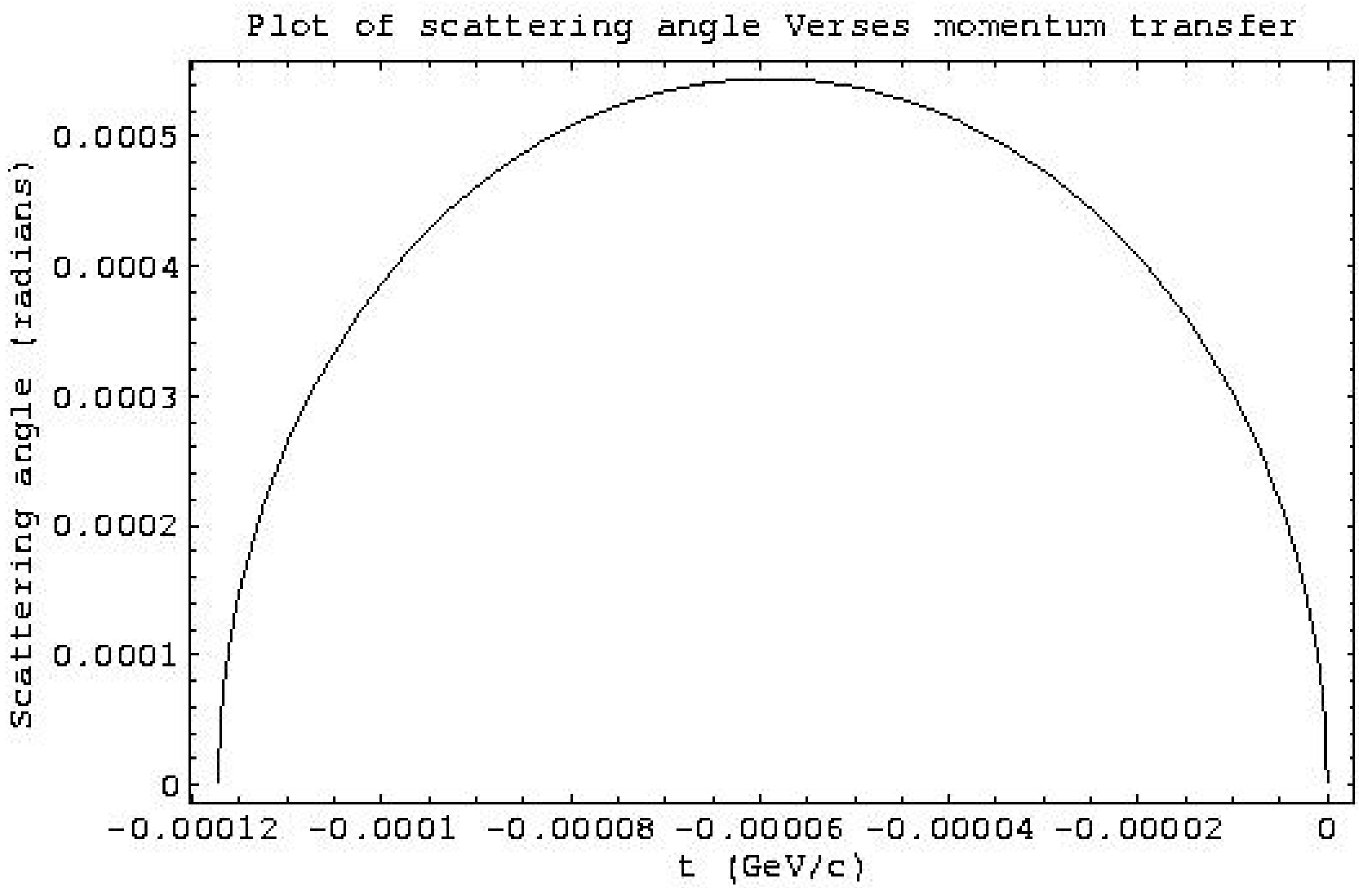
$$(1 - D_{\rm ZZ})\,\frac{d\,\sigma}{d\,\Omega} \approx \frac{-M^2\,\alpha^2\,\left(s + m^2 - M^{\,2}\right)^2}{2\; k^2\; s^2\; t}$$

3 The $\mathcal{PAX}$ Project

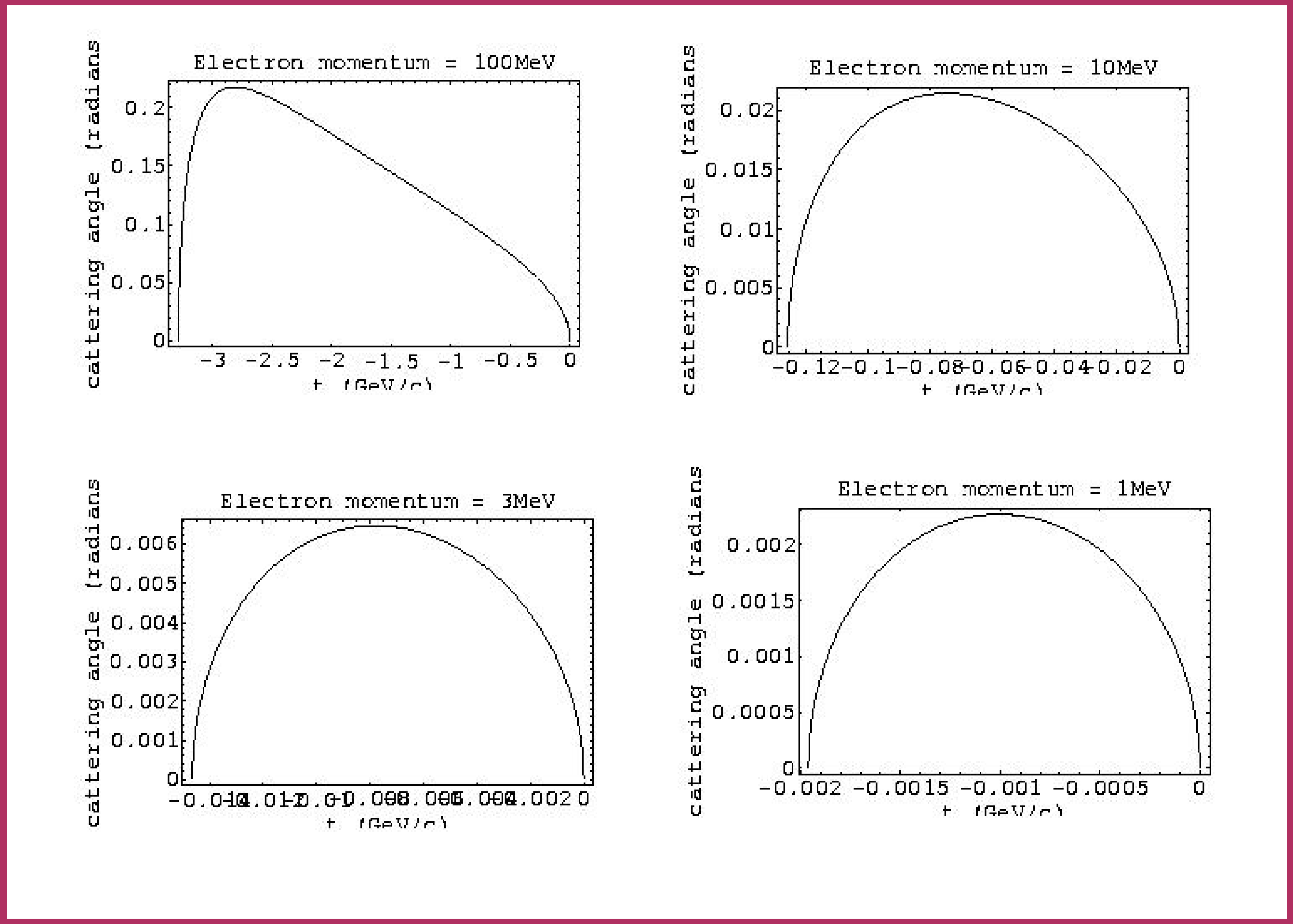
- There has been much recent debate as to whether electrons in the hydrogen target will transfer polarization to the antiproton beam.

- We're investigating if a beam of polarized electrons with sufficiently high density could be used to polarize an antiproton beam.

3.1 Scattering is within the ring for stationary electrons



3.2 Effect of electron beam momentum



4 Polarization buildup

When circulating at frequency ν through a polarized target of areal density n and polarization P_e oriented normal to the ring plane,

$$\frac{d}{dt}\begin{bmatrix} N \\ J \end{bmatrix} = -n\,\nu \begin{bmatrix} I_{\text{out}} & P_e\,A_{\text{out}} \\ P_e\,A_{\text{all}} - P_e\,K_{\text{in}} & I_{\text{all}} - D_{\text{in}} \end{bmatrix} \begin{bmatrix} N \\ J \end{bmatrix}$$

describes the rate of change of the number of beam particles N and their total spin J.

These coupled differential equations involve angular integration of the spin observables presented earlier.

Transverse polarization requires	Longitudinal polarization requires
$I_{\rm out} = 2\pi \int_{\theta_{\rm acc}}^{\pi} \frac{d\sigma}{d\Omega} \sin\theta\, d\theta$	$I_{\rm out} = 2\pi \int_{\theta_{\rm acc}}^{\pi} \frac{d\sigma}{d\Omega} \sin\theta\, d\theta$
$A_{\rm out} = \pi \int_{\theta_{\rm acc}}^{\pi} (A_{\rm XX} + A_{\rm YY}) \frac{d\sigma}{d\Omega} \sin\theta\, d\theta$	$A_{\rm out} = 2\pi \int_{\theta_{\rm acc}}^{\pi} A_{\rm LL} \frac{d\sigma}{d\Omega} \sin\theta\, d\theta$
$A_{\rm all} = \pi \int_{\theta_0}^{\pi} (A_{\rm XX} + A_{\rm YY}) \frac{d\sigma}{d\Omega} \sin\theta\, d\theta$	$A_{\rm all} = 2\pi \int_{\theta_0}^{\pi} A_{\rm LL} \frac{d\sigma}{d\Omega} \sin\theta\, d\theta$
$K_{\rm in} = \pi \int_{\theta_0}^{\theta_{\rm acc}} (K_{\rm XX} + K_{\rm YY}) \frac{d\sigma}{d\Omega} \sin\theta\, d\theta$	$K_{\rm in} = 2\pi \int_{\theta_0}^{\theta_{\rm acc}} K_{\rm LL} \frac{d\sigma}{d\Omega} \sin\theta\, d\theta$
$D_{\rm in} = \pi \int_{\theta_0}^{\theta_{\rm acc}} (D_{\rm XX} + D_{\rm YY}) \frac{d\sigma}{d\Omega} \sin\theta\, d\theta$	$D_{\rm in} = 2\pi \int_{\theta_0}^{\theta_{\rm acc}} D_{\rm LL} \frac{d\sigma}{d\Omega} \sin\theta\, d\theta$

4.1 Solution of the system

The time dependence of the polarization of the beam is given by solving the coupled system of differential equations, leading to

$$P(t) = \frac{J(t)}{N(t)} = -P_{\mathrm{e}} \frac{A_{\mathrm{all}} - K_{\mathrm{in}}}{L_{\mathrm{in}} + L_{\mathrm{d}} \coth(L_{\mathrm{d}}\, n\, \nu\, t)}$$

where the discriminant of the quadratic equation for the eigenvalues is

$$L_{\mathrm{d}} = \sqrt{P_{\mathrm{e}}^2\, A_{\mathrm{out}} (A_{\mathrm{all}} - K_{\mathrm{in}}) + L_{\mathrm{in}}^2}$$

and $L_{\mathrm{in}} = (I_{\mathrm{in}} - D_{\mathrm{in}}) / 2$ is the loss of polarization quantity. For sufficiently short times, the rate of change of polarization is approximately

$$\frac{dP}{dt} \approx -n\, \nu\, P_{\mathrm{e}} (A_{\mathrm{all}} - K_{\mathrm{in}})$$

and the limit of the polarization for large times is

$$\lim_{t \to \infty} P(t) = -P_{\mathrm{e}} \frac{A_{\mathrm{all}} - K_{\mathrm{in}}}{L_{\mathrm{in}} + L_{\mathrm{d}}}\,.$$

Further work

- Complete this analyses and obtain a numerical estimate for the polarization buildup rate, with a hydrogen gas target and also with an electron beam.
- Similarly calculate all electromagnetic helicity amplitudes and spin observables for antiproton-deuteron scattering. Estimate the polarization buildup rate for antiproton scattering off a polarized deuteron target.

References

Antiproton polarization has been considered recently by F. Rathmann *et al.* (2005), N. N. Nikolaev *et al.* (2006), A. I. Milstein *et al.* (2005) and T. Walcher *et al.* (2006).

Results are consistent with the earlier work of B. Z. Kopeliovich and L. I. Lapidus (1974); N. H. Buttimore, E. Gotsman and E. Leader (1978), P. La France and P. Winternitz (1980), J. Bystricky, F. Lehar and P. Winternitz (1978) and E. Leader (2005).

Conclusions

- All Helicity Amplitudes and Spin Observables for elastic spin 1/2 - spin 1/2 scattering have been presented to first order in QED.
- A beam of polarized electrons could be used to increase the polarization of an antiproton beam by spin filtering.
- Using the spin observables a numerical estimate for the rate of build up of polarization of an antiproton beam is being obtained for the $\mathcal{PAX}$ project.

Queries to email: donie@maths.tcd.ie

Acknowledgments

I thank "The Embark Initiative" (IRCSET), for a postgraduate research scholarship.