Spin Observables and Antiproton Polarisation

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Introduction

- Relativistic formulae for spin dependent (polarisation transfer) one photon exchange differential cross sections are developed for spin $1/2$ fermion-fermion elastic scattering.

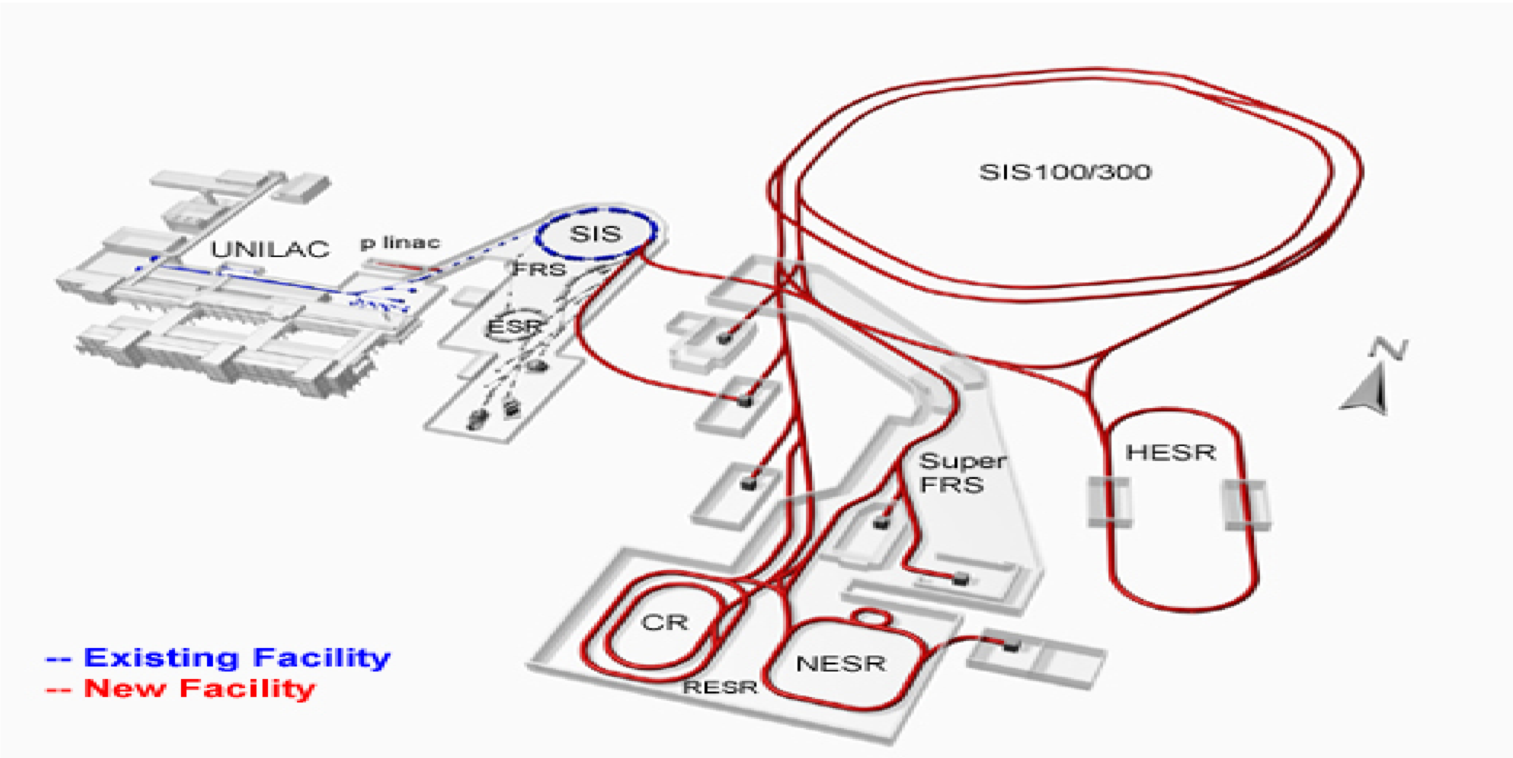
- The Polarised Antiproton eXperiments (PAX) project at GSI Darmstadt, require spin observables for polarising an antiproton beam.

- In particular, cross sections for polarisation transfer in antiproton-electron $\bar{p}\,e\!\uparrow \longrightarrow \bar{p}\!\uparrow e$ and antiproton-proton $\bar{p}\,p\!\uparrow \longrightarrow \bar{p}\!\uparrow p$ elastic scattering are needed, for interaction with a polarised Hydrogen target.

- Helicity amplitudes are then derived and used to determine the spin observables of the reactions.

The Future GSI Facility

The Future Accelerator Layout



1 Normalisation

The differential cross section is related to the amplitude $\mathcal{M}$ by

$$s\,\frac{d\sigma}{d\Omega} = \frac{1}{(8\pi)^2} \sum_{\lambda\lambda'\Lambda\Lambda'} \frac{1}{(2\,s+1)(2\,S+1)}\,|\mathcal{M}|^2$$

The electron current is

$$j^\mu = e\,\bar{u}(k',\lambda')\,\gamma^\mu\,u(k,\lambda)$$

and the proton current, after Gordon decomposition is

$$J_\mu = e\,\bar{u}(P',\Lambda')\left(G_M\,\gamma_\mu - F_2\,\frac{P_\mu + P'_\mu}{2M}\right)u(P,\Lambda)$$

1.1 Spin Averaged Case

Define electromagnetic form factors $F_1(q^2)$ and $F_2(q^2)$ such that $F_1(0) = 1$ and $F_2(0) = \mu - 1$, the anomalous magnetic moment, and for convenience use the Sachs form factors $G_M = F_1 + F_2$ and $G_E = F_1 + F_2\, t/4\, M^2$ with the corresponding lower case form factors for the second particle. Thus the differential cross-section for one photon exchange, of two non-identical spin 1/2 fermions is

$$\frac{s}{\alpha^2}\frac{d\sigma}{d\Omega} = \left(\frac{4\,m^2\,g_E^2 - t\,g_M^2}{4\,m^2 - t}\right)\left(\frac{4\,M^2\,G_E^2 - t\,G_M^2}{4\,M^2 - t}\right)\frac{\left(M^2 - m^2\right)^2 - s\,u}{t^2}$$

$$+\left(\frac{2\,m\,M\,g_E\,G_E}{t}\right)^2 + \frac{1}{2}\,g_M^2\,G_M^2$$

where m and M are the masses of the particles, the s, t and u are the Mandelstam variables, and $\alpha = e^2/4\pi$, the QED coupling constant.

1.2 Antiproton Proton Case

For antiproton-proton collisions, the electromagnetic form factors and masses of the proton and antiproton are the same, i.e. $f_1 = F_1$ and $f_2 = F_2$ so $g_M = G_M$; and $m = M$. Here we neglect the s-channel one photon contribution in favour of the t-channel term which dominates in the low momentum transfer (small t) region of interest. This gives

$$\frac{s}{\alpha^2}\frac{d\sigma}{d\Omega} = \left(\frac{4\,M^{\,2}\,G_E^{\,2} - t\,G_M^{\,2}}{4\,M^{\,2} - t}\right)^2 \left(\frac{-s\,u}{t^2}\right) + \left(\frac{2\,M^{\,2}\,G_E^{\,2}}{t}\right)^2 + \frac{1}{2}\,G_M^{\,4}$$

This agrees with an expression formed from known fermion fermion helicity amplitudes (Buttimore *et al.*, Phys. Rev. D **18** (1978) 694) and (Winternitz *et al.*, J. Phys. (Paris) **41** (1980) 1391).

This result is important in the momentum transfer region $|t| < |t_c|$ for antiproton proton collisions with total cross section $\sigma_{\rm tot}$, defined by

$$t_c = -\frac{8\,\pi\,\alpha}{\beta_{\rm lab}\,\sigma_{\rm tot}} \approx -\,0.001\,(\mathsf{GeV}/c)^2$$

where the electromagnetic interaction dominates the hadronic interaction.

Here $\beta_{\rm lab}$ is the laboratory velocity.

See DOB and N. H. Buttimore, Czech. J. Phys. **56** (2006); hep-ph/0605099.

2 Spin Transfer Cases

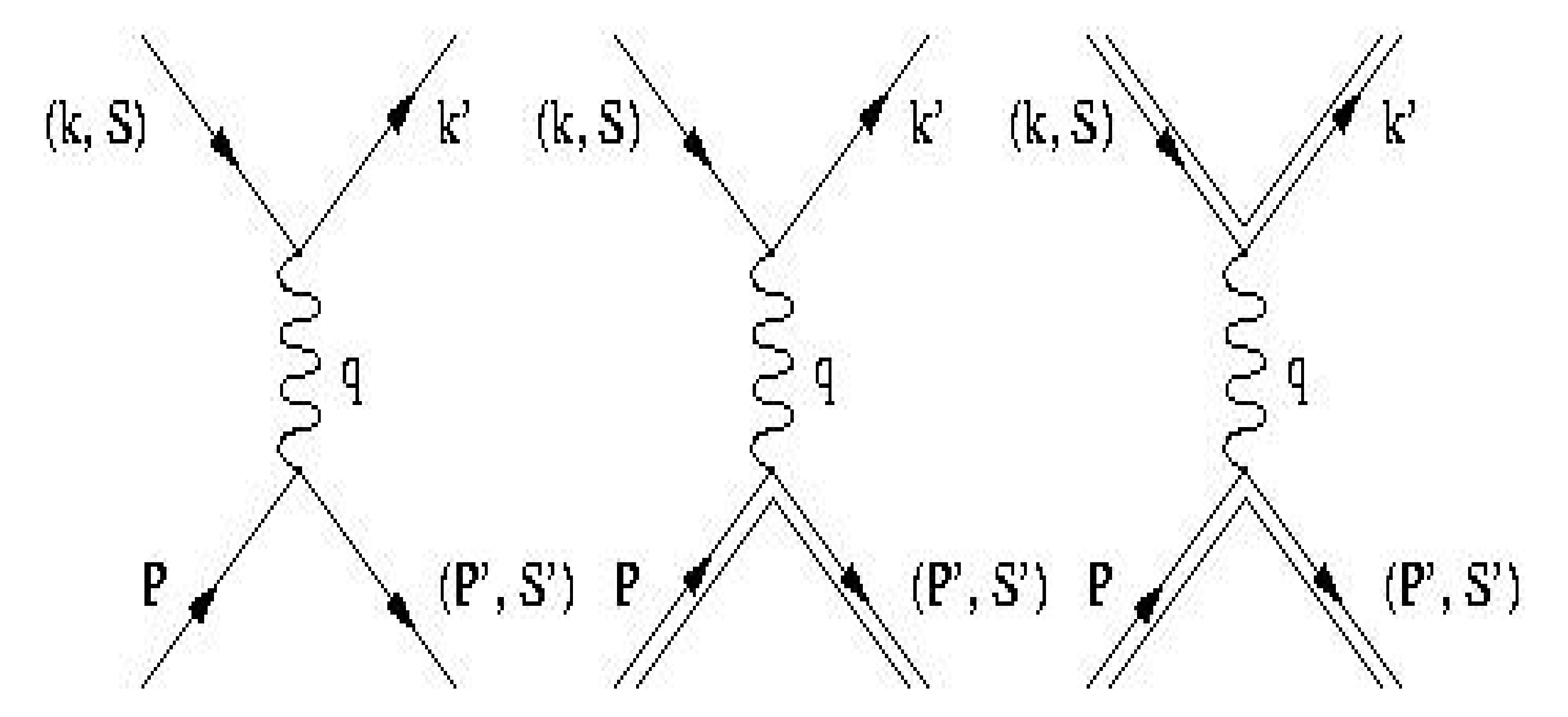


Figure: Feynman diagrams for the three spin dependent cases.

2.1 Notation

- Suppose the initial electron (or proton) to have a spin four vector S_μ and the final scattered antiproton to have a spin four vector S'_μ.

- We are interested in polarisation transfer K_{j00i}, $\bar{p}\,p\!\uparrow \longrightarrow \bar{p}\!\uparrow p$.

$$\bar{p}(P) + p(k, S) \longrightarrow \bar{p}(P', S') + p(k')$$

- Consider just the spin dependent terms, and use the notation $A \cdot B = A_\mu B^\mu$.

- Define the momentum transfer to the photon $q = k' - k = P - P'$, and use $q^2 = t$. The spin vectors are normalised so that $S^\mu S_\mu = -1$.

2.2 Structureless

The cross section for polarisation transfer K_{j00i}, from initial electron to final antiproton (assumed structureless here) is

$$s\,\frac{d\sigma}{d\Omega}\,K_{j00i} = -\left(\frac{2\,\alpha^2}{t}\right) mM \left[S\cdot S' - \frac{S\cdot q\, S'\cdot q}{t} \right] .$$

Note the singular $1/t$ dependence, which make this large in the small t region of interest.

2.3 One Particle Structured

Using the proton electromagnetic form factors as described earlier, and also $S \cdot k = 0$ and $S' \cdot P' = 0$ from the general theory of spin polarisation.

$$
\begin{aligned}
s \frac{d\sigma}{d\Omega} K_{j00i} \quad &= \quad -\left(\frac{2\,\alpha^2}{t}\right) mMG_M \left\{ F_1 \left[S \cdot S' - \frac{S \cdot q\, S' \cdot q}{t} \right] \right. \\
&\qquad \left. + \frac{F_2}{4M^2} \left[t\, S \cdot S' + 2\, S \cdot P'\, S' \cdot q \right] \right\}
\end{aligned}
$$

This result reduces to that of the previous section in the limit $F_1 \to 1$, $F_2 \to 0$ and hence $G_M \to 1$.

The above expression represents a relativistic generalisation of equation (3) of Horowitz & Meyer, PRL **72** (1994) 3981.

2.4 Antiproton Proton Case

Again the proton and antiproton electromagnetic form factors and masses are the same, and the s-channel term is neglected in favour of the t-channel contribution in the low momentum transfer region of interest. Thus

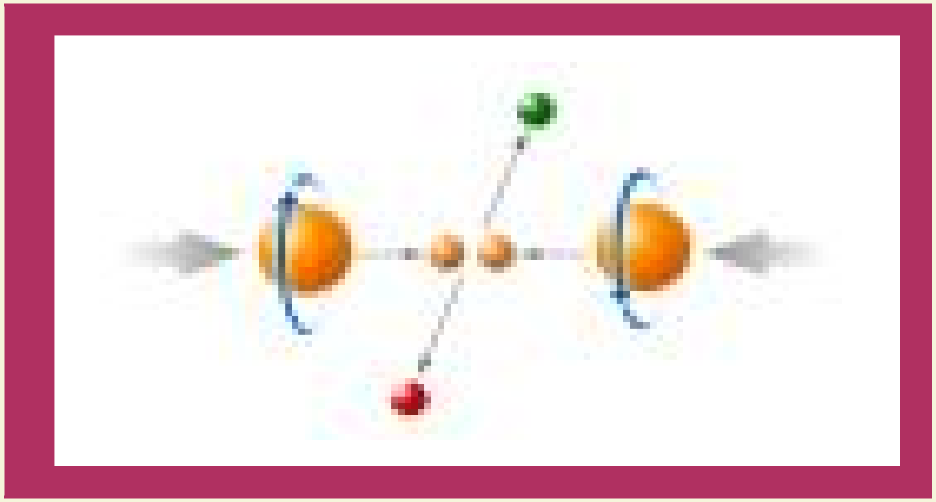
$$s\,\frac{d\sigma}{d\Omega}\,K_{j00i} = -\,\alpha^2\,G_M^2\,\Bigg\{\frac{F_2^2}{8\,M^2\,t}\,\big(\,t + 2\,s - \,4\,M^2\big)\;q\cdot S_1\;q\cdot S_4$$

$$+\,G_E\left[\frac{F_2}{2\,t}\,(\,p_1 + p_3\,)\cdot S_4\;q\cdot S_1 + \left(\frac{F_2}{t}\,p_4\cdot S_1 - \frac{2\,F_1}{t^2}\,M^2\,q\cdot S_1\right)\,q\cdot S_4\right]$$

$$+\,\frac{2\,M^2}{t}\,G_E^2\,S_1\cdot S_4\,\Bigg\}$$

and again this result is important in the momentum transfer region $|t| < |t_c|$ defined earlier, where the electromagnetic interaction dominates the hadronic interaction.

3 The EM Helicity Amplitudes and Spin Observables



3.1 The Generic Calculation

The generic equation for polarisation effects in spin 1/2 - spin 1/2 scattering to first order in QED is

$$16\,(q/e)^4\,|\mathcal{M}|^2 =$$

$$\mathrm{Tr}\left[\left(\not{p}_1 + m\right)(1+\gamma_5\,\not{S}_1)\,(g_M\gamma^\nu + f r^\nu)\left(\not{p}_3 + m\right)(1+\gamma_5\,\not{S}_3)\,(g_M\gamma^\mu + f r^\mu)\right]\times$$

$$\mathrm{Tr}\left[\left(\not{p}_4 + M\right)(1+\gamma_5\,\not{S}_4)\,(G_M\gamma_\mu + F R_\mu)\left(\not{p}_2 + M\right)(1+\gamma_5\,\not{S}_2)\,(G_M\gamma_\nu + F R_\nu)\right]$$

where $r^\mu = p_1^\mu + p_3^\mu$, $R^\mu = p_2^\mu + p_4^\mu$, $F = -\,F_2/2\,M$ and $f = -\,f_2/2\,m$. This generic equation can thus be used to calculate all helicity amplitudes and spin observables etc. by substituting specific values for the spin (S_i) and momenta (p_i) vectors. The result has been obtained for this equation with the traces computed and contracted, using *Mathematica*.

3.2 Momentum and Spin Vectors

In the Centre-of-Mass (CM) frame the momenta and longitudinal (helicity) spin vectors are

$$
\begin{array}{lcl|lcl}
p_1 & = & (\,E_1,\,0,\,0,\,k) & p_3 & = & (\,E_1,\,k\,\sin\theta,\,0,\,k\,\cos\theta) \\
p_2 & = & (\,E_2,\,0,\,0,\,-k) & p_4 & = & (\,E_2,\,-k\,\sin\theta,\,0,\,-k\,\cos\theta) \\
S_1 & = & \frac{1}{M_1}\,(\,k,\,0,\,0,\,E_1) & S_3 & = & \frac{1}{M_1}(\,k,\,E_1\,\sin\theta,\,0,\,E\,\cos\theta) \\
S_2 & = & -\frac{1}{M_2}\,(\,-k,\,0,\,0,\,E_2) & S_4 & = & -\frac{1}{M_2}(\,-k,\,E_2\,\sin\theta,\,0,\,E\,\cos\theta)
\end{array}
$$

The notation of the helicily amplitudes $H(\,A',B'\,;\,A,B\,)$ is $H(\,\pm,\pm\,;\,\pm,\pm\,)$ where the arguments are $+$ if the spin vector is as above (polarised along the direction of motion) and $-$ if the spin vector is minus the above (polarised opposite to the direction of motion).

3.3 Helicity Amplitudes

After using T and P invariance there are 6 independent helicity amplitudes for the scattering of two non-identical spin 1/2 particles.

$$
\begin{aligned}
H(+,+;+,+) &\equiv \phi_1 \\
H(+,+;-,-) &\equiv \phi_2 \\
H(+,-;+,-) &\equiv \phi_3 \\
H(+,-;-,+) &\equiv \phi_4 \\
H(+,+;+,-) &\equiv \phi_5 \\
H(+,+;-,+) &\equiv \phi_6
\end{aligned}
$$

Note for $p\,p$, $\bar{p}\,p$ and $\bar{p}\,\bar{p}$ scattering $\phi_6 = -\phi_5$, so there are only 5 independent helicity amplitudes.

3.4 First order QED results

$$\frac{\phi_1}{\alpha} = \frac{s-m^2-M^2}{t}\left(1+\frac{t}{4k^2}\right) f_1 F_1 - f_1 F_1 - f_2 F_1 - f_1 F_2 - \frac{1}{2} f_2 F_2 \left(1-\frac{t}{4k^2}\right)$$

$$\frac{\phi_2}{\alpha} = \frac{1}{2}\left(\frac{m}{k} f_1 - \frac{k}{m} f_2\right)\left(\frac{M}{k} F_1 - \frac{k}{M} F_2\right) + \frac{s-m^2-M^2-2k^2}{4mM}\left(1+\frac{t}{4k^2}\right) f_2 F_2$$

$$\frac{\phi_3}{\alpha} = \left[\frac{s-m^2-M^2}{t} f_1 F_1 + \frac{f_2 F_2}{2}\right]\left(1+\frac{t}{4k^2}\right)$$

$$\phi_4 = -\phi_2$$

$$\frac{\phi_5}{\alpha} = \sqrt{\frac{s}{-t}(4k^2+t)}\left[\frac{f_1 F_1 M}{4k^2}\left(1+\frac{m^2-M^2}{s}\right) - \frac{f_1 F_2}{2M} + \frac{t f_2 F_2}{16 M k^2}\left(1+\frac{M^2-m^2}{s}\right)\right]$$

$$\frac{\phi_6}{\alpha} = \sqrt{\frac{s}{-t}(4k^2+t)}\left[\frac{-f_1 F_1 m}{4k^2}\left(1+\frac{M^2-m^2}{s}\right) + \frac{f_2 F_1}{2m} - \frac{t f_2 F_2}{16 m k^2}\left(1+\frac{m^2-M^2}{s}\right)\right]$$

The combinations $\phi_1 + \phi_3$ and $\phi_1 - \phi_3$ appear often in the observables

$$\frac{\phi_1 + \phi_3}{\alpha} = -g_M\, G_M + \left(1 + \frac{t}{4\,k^2}\right) \left[\frac{s - m^2 - M^2}{t}\, 2\, f_1\, F_1 + f_2\, F_2\right]$$

$$\frac{\phi_1 - \phi_3}{\alpha} = -g_M\, G_M$$

All the spin observables of a reaction (polarisation transfer, depolarisation and assymetries) can be written as bilinear combinations of the helicity amplitudes. So they can now be easily obtained. Numerical results require specific values for the masses and form factors of each particle, the CM momenta k and t.

4 The polarisation transfer spin observables

To find the spin observable K_{NN}, the normal polarised spin vectors $S_1 = (\,0,\,0,\,1,\,0\,)$ and $S_4 = (\,0,\,0,\,1,\,0\,)$ and the CM momenta vectors, are inputted into the generic equation. This gives

$$s\,\frac{d\sigma}{d\Omega}\,K_{NN} = \left(\frac{2\,\alpha^2}{t}\right)\;m\,M\,g_E\,g_M\,G_E\,G_M$$

where $g_E = f_1 + \frac{t}{4\,m^2}\,f_2$. When the longitudinal polarised spin vectors mentioned earlier are used we get the spin observable K_{LL}

$$\begin{aligned} s\,\frac{d\sigma}{d\Omega}\,K_{LL} \;\;=\;\; & -\alpha^2\,\frac{g_M\,G_M}{8\;k^2\;t}\,\Big\{4\,\sqrt{k^2+m^2}\,\sqrt{k^2+M^{\,2}}\;\left(4\,k^2+t\right)\;f_1\,F_1 \\ & + \left(4\,k^2\,f_1 - t\,f_2\right)\;\left(\,4\,k^2\,F_1 - t\,F_2\,\right)\Big\} \end{aligned}$$

When the transverse polarised spin vectors $S_1 = (\,0,\,1,\,0,\,0\,)$ and $S_4 = (\,0,\,\cos\theta,\,0,\,-\sin\theta\,)$ are inputted into the generic equation we get

$$
\begin{aligned}
s\,\frac{d\sigma}{d\Omega}\,K_{SS} \quad = \quad & \alpha^2 \frac{g_M\,G_M}{8\,k^2\,m\,M}\,\Big\{\,4\,m^2\,f_1\,\left(M^{\,2}\,F_1 - k^2\,F_2\right) + f_2\,\Big[-4\,k^2\,M^{\,2}\,F_1 \\
& + \left(4\,k^4 + \left(4\,k^2\,+t\right)\sqrt{k^2+m^2}\,\sqrt{k^2+M^{\,2}}\right)\,F_2\Big]\Big\}
\end{aligned}
$$

When $S_1 = (\,0,\,1,\,0,\,0\,)$ and $S_4 = \frac{1}{M}\,(-k,\,E_1\,\sin\theta,\,0,\,E_1\,\cos\theta)$ are inputted, where $E_1 = \sqrt{k^2+M^2}$, we obtain

$$
\begin{aligned}
s\,\frac{d\sigma}{d\Omega}\,K_{SL} \quad = \quad & \frac{\alpha^2\,g_M\,G_M}{8\,m\,t}\sqrt{\frac{-\,t\,\left(\,4\,k^2+t\,\right)}{k^{\,4}}}\,\Big\{\,4\,m^2\,\sqrt{k^2+M^{\,2}}\,f_1\,F_1 \\
& - f_2\,\Big[\,4\,k^2\,\sqrt{s}\,F_1 - \sqrt{k^2+m^2}\;t\,F_2\Big]\Big\}
\end{aligned}
$$

When the spin four vectors $S_1 = \frac{1}{m}\,(\,k,\,0,\,0,\,E_2\,)$ and $S_4 = -\,(\,0,\,\cos\theta,\,0,\,-\sin\theta\,)$ are inputted, where $E_2 = \sqrt{k^2+m^2}$, we obtain

$$s\,\frac{d\sigma}{d\Omega}\,K_{LS} \;=\; \frac{\alpha^2\,g_M\,G_M}{8\,M\,t}\sqrt{\frac{-\,t\;(\,4\,k^2+t\,)}{k^{\,4}}}\;\Big\{\sqrt{k^2+M^{\,2}}\,t\,f_2\,F_2$$

$$+\,4\,f_1\,\Big[\,\sqrt{k^2+m^2}\;M^{\,2}\,F_1 - k^2\,\sqrt{s}\,F_2\Big]\Big\}$$

In all the spin observables above k can be eliminated in favour of s. Also

$$K_{SN} = K_{NS} = K_{LN} = K_{NL} = 0$$

These are all in the Centre-of-Mass frame, we have similar expressions in the LAB frame. These simplify for the case of structureless particles and for electron scattering the approximation $m \rightarrow 0$ is often made.

4.1 The $p\,p$, $\bar{p}\,p$ and $\bar{p}\,\bar{p}$ cases.

For $p\,p$, $\bar{p}\,p$ and $\bar{p}\,\bar{p}$ elastic scattering the masses and form factors of both particles are the same. The helicity amplitudes are

$$\frac{\phi_1}{\pm\,\alpha} = \left(\frac{s+4\,k^2}{2\,t}+\frac{M^{\,2}}{2\,k^2}\right)F_1^{\,2}-2\,F_1\,F_2+\left(\frac{t-4\,k^2}{8\,k^2}\right)F_2^{\,2}$$

$$\frac{\phi_2}{\pm\,\alpha} = \left(\frac{M^{\,2}}{2\,k^2}\right)F_1^{\,2}-F_1\,F_2+\left(\frac{t-4\,k^2}{8\,k^2}+\frac{2\,s+t}{8\,M^{\,2}}\right)F_2^{\,2}$$

$$\frac{\phi_3}{\pm\,\alpha} = \left[\frac{s-2\,M^{\,2}}{t}\,F_1^{\,2}+\frac{F_2^{\,2}}{2}\right]\left(1+\frac{t}{4\,k^2}\right)$$

$$\phi_4 = -\phi_2$$

$$\frac{\phi_5}{\pm\,\alpha} = \frac{1}{2\,M}\sqrt{\frac{s\,u}{t}}\left[\left(\frac{M^{\,2}}{2\,k^2}\right)F_1^{\,2}-F_1\,F_2+\left(\frac{t}{8\,k^2}\right)F_2^{\,2}\right]$$

where $\phi_6=-\phi_5$ in this case.

The factor of $\pm$ difference between here and earlier is from the opposite charge of the antiproton $e_p = -e_{\bar{p}}$ so $e_p\, e_{\bar{p}} = -e_p^2$. So the $\pm$ above are $+$ for $p\,p$ and $\bar{p}\,\bar{p}$ scattering and $-$ for $\bar{p}\,p$ scattering. Thus the spin transfer observables now become

$$s\,\frac{d\sigma}{d\Omega}\,K_{NN} \;=\; \left(\frac{2\,\alpha^2}{t}\right)\,M^2\,G_E^2\,G_M^2$$

$$s\,\frac{d\sigma}{d\Omega}\,K_{LL} \;=\; \frac{-\alpha^2\,G_M^2}{8\;k^2\;t}\left\{s\,(4\,k^2+t)\,F_1^2 + \left(4\,k^2\,F_1 - t\,F_2\right)^2\right\}$$

$$s\,\frac{d\sigma}{d\Omega}\,K_{SS} \;=\; \frac{\alpha^2\,G_M^2}{8\,k^2\,M^2}\left\{4\,M^4\,F_1^2 \;-\; 8\,k^2\,M^2\,F_1\,F_2\right.$$

$$\left. +\left(4\,k^4 + \left(k^2+\frac{t}{4}\right)s\right)\,F_2^2\right\}$$

For $p\,p$, $\bar{p}\,p$ and $\bar{p}\,\bar{p}$ elastic scattering $K_{LS} = K_{SL}$ thus we obtain

$$s\,\frac{d\sigma}{d\Omega}\,K_{LS} = s\,\frac{d\sigma}{d\Omega}\,K_{SL} =$$

$$= \frac{\alpha^2\,G_M^2}{16\,M\,t}\sqrt{\frac{-\,t\,s\,\left(\,4\,k^2 + t\,\right)}{k^{\,4}}}\;\left\{\,t\,F_2^2 + 4\,F_1\,\left[\,M^{\,2}\,F_1 - 2\,k^2\,F_2\,\right]\right\}$$

5 The PAX Project

PAX

Polarized Antiproton EXperiments

5.1 The PAX Proposal

- The PAX project proposes to build up the polarisation of an antiproton beam by repeated interaction with a polarised Hydrogen target in a storage ring.
- A numerical estimate for the build up of polarisation of the antiproton beam can be obtained from our results.
- We must consider the antiproton interactions with the polarised electron and the polarised proton inside the Hydrogen target.
- The most important spin observables are the polarisation transfer from electron or proton in the target to the antiproton beam (K_{j00i}), the depolarisation of the antiproton beam (D_{0j0i}) and the double spin asymmetry (A_{00ij}).

3.2 Polarisation buildup

When circulating at frequency ν through a polarised target of areal density n and polarisation P oriented normal to the ring plane,

$$\frac{d}{dt}\begin{bmatrix} I_0 \\ J \end{bmatrix} = -n\,\nu \begin{bmatrix} I_{\text{out}} & P\,A_{\text{out}} \\ P\,A_{\text{all}} - P\,K_{\text{in}} & I_{\text{all}} - D_{\text{in}} \end{bmatrix} \begin{bmatrix} I_0 \\ J \end{bmatrix}$$

describes the rate of change of the number of beam particles I_0 and their total spin J. The loss of particles from the beam involves

$$I_{\text{out}} = 2\,\pi \int_{\theta_{\text{acc}}}^{\pi} \frac{d\,\sigma}{d\,\Omega}\,\sin\theta\,d\theta$$

as coefficient, where integration is over all angles beyond θ_{acc}, the acceptance angle of the accelerator ring.

A further change in the beam results from a product of the target polarisation P with the azimuthal average of the transverse double spin asymmetry

$$A_{\rm out} = \pi \int_{\theta_{\rm acc}}^{\pi} (A_{\rm NN} + A_{\rm SS}) \frac{d\sigma}{d\Omega} \sin\theta \, d\theta$$

again integrated over collision angles beyond the acceptance angle.
A corresponding change in the total spin J involves a product of P with the average transverse asymmetry integrated over all angles

$$A_{\rm all} = \pi \int_{\theta_0}^{\pi} (A_{\rm NN} + A_{\rm SS}) \frac{d\sigma}{d\Omega} \sin\theta \, d\theta$$

above a minimum angle θ_0 associated with the Bohr radius of a hydrogen atom, an impact parameter beyond which scattering is inhibited. The sum

$$A_{\rm all} = A_{\rm in} + A_{\rm out}.$$

Another contribution to the change in J comes from P times the azimuthally averaged spin transfer observable, below acceptance

$$K_{\rm in} = \pi \int_{\theta_0}^{\theta_{\rm acc}} (K_{\rm NN} + K_{\rm SS}) \frac{d\,\sigma}{d\,\Omega}\, \sin\theta\, d\theta$$

when integrated over low angles of scattering corresponding to particles remaining in the ring.

A contribution to a change in J involving J itself results from the azimuthally averaged depolarisation observable, below acceptance

$$D_{\rm in} = \pi \int_{\theta_0}^{\theta_{\rm acc}} (D_{\rm NN} + D_{\rm SS}) \frac{d\,\sigma}{d\,\Omega}\, \sin\theta\, d\theta$$

It is understood that electron and proton contributions should be summed incoherently.

5.3 Numerical results

Preliminary results show that the spin observables for antiproton electron scattering near threshold ($s = 4\,M^2$) and at $t = t_c = -0.001\,(\text{GeV}/c)^2$ are:

$$
\begin{aligned}
A_{NN} &= K_{NN} \approx -\,0.11\,\% \\
A_{SS} &= K_{SS} \propto t_c^2 \qquad \text{so is negligible}
\end{aligned}
$$

Recent work also has been done on this by N. N. Nikolaev *et al.* (2006), A. I. Milstein *et al.* (2005) and T. Walcher *et al.* (2006).
Our results are consistent with the earlier work of B. Z. Kopeliovich and L. I. Lapidus (1974), N. H. Buttimore, E. Gotsman and E. Leader (1978), P. La France and P. Winternitz (1980) and J. Bystricky, F. Lehar and P. Winternitz (1978).

Conclusions

- Relativistic differential cross section formulae have been derived for polarisation transfer in spin 1/2 fermion-fermion elastic scattering, due to one photon exchange.
- Helicity Amplitudes for spin 1/2 fermion-fermion elastic scattering have been calculated and can be used to determine the relevant spin observables of a reaction.
- A numerical estimate for the rate of build up of polarisation of an antiproton beam is being obtained for the PAX project.

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Extra Slides

Polarisation Buildup

The time dependence of the polarised beam is given by solving the coupled system of differential equations, leading to

$$P(t) \;=\; P_{\rm e}\,\frac{A_{\rm all}\;-\;K_{\rm in}}{L_{\rm out}\;+\;L_{\rm dsc}\,\coth\left(n\,\nu\,L_{\rm dsc}\right)}$$

where the discriminant of the quadratic equation for the eigenvalues is

$$L_{\rm dsc} \;=\; \sqrt{4\,P_{\rm e}^2\,A_{\rm out}\left(A_{\rm all}\;-\;K_{\rm in}\right)\;+\;L_{\rm in}^2}\,.$$

and

$$L_{\rm in} \;=\; I_{\rm in}\;-\;D_{\rm in}$$

is the loss of polarisation quantity. For sufficiently short times, the rate of change of polarisation is approximately

$$\frac{dP}{dt} \approx n\,\nu\,P_{\rm e}\,L_{\rm dsc}\,\frac{K_{\rm in}\;-\;A_{\rm all}}{L_{\rm dsc}\;+\;L_{\rm out}}\,.$$

In the case of proton proton elastic scattering at a laboratory kinetic energy of 23 MeV Meyer estimates from the SAID database that the purely hadronic double spin asymmetry parameter has the value

$$A^{\mathrm{pp}}_{\mathrm{all}} = 122 \ \mathrm{mb}$$

and what is more relevant, that for collisions due to electromagnetic and hadronic interactions beyond the acceptance angle the double asymmetry parameter is

$$A^{\mathrm{pp}}_{\mathrm{out}} = 83 \ \mathrm{mb}$$

while the purely hadronic $A^{\mathrm{pp}}_{\mathrm{in}}$ is negligible.

Noting that all proton electron scattering occurs within the acceptance angle, Horowitz and Meyer evaluated the polarisation transfer parameter due to proton electron elastic scattering as

$$K_{\rm in}^{\rm pe} = -70 \ {\rm mb}$$

and the asymmetry parameter resulting from elastic proton proton collisions below the acceptance angle

$$K_{\rm in}^{\rm pp} = 52 \, {\rm mb}$$