Spin dependent elastic antiproton interactions

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Introduction

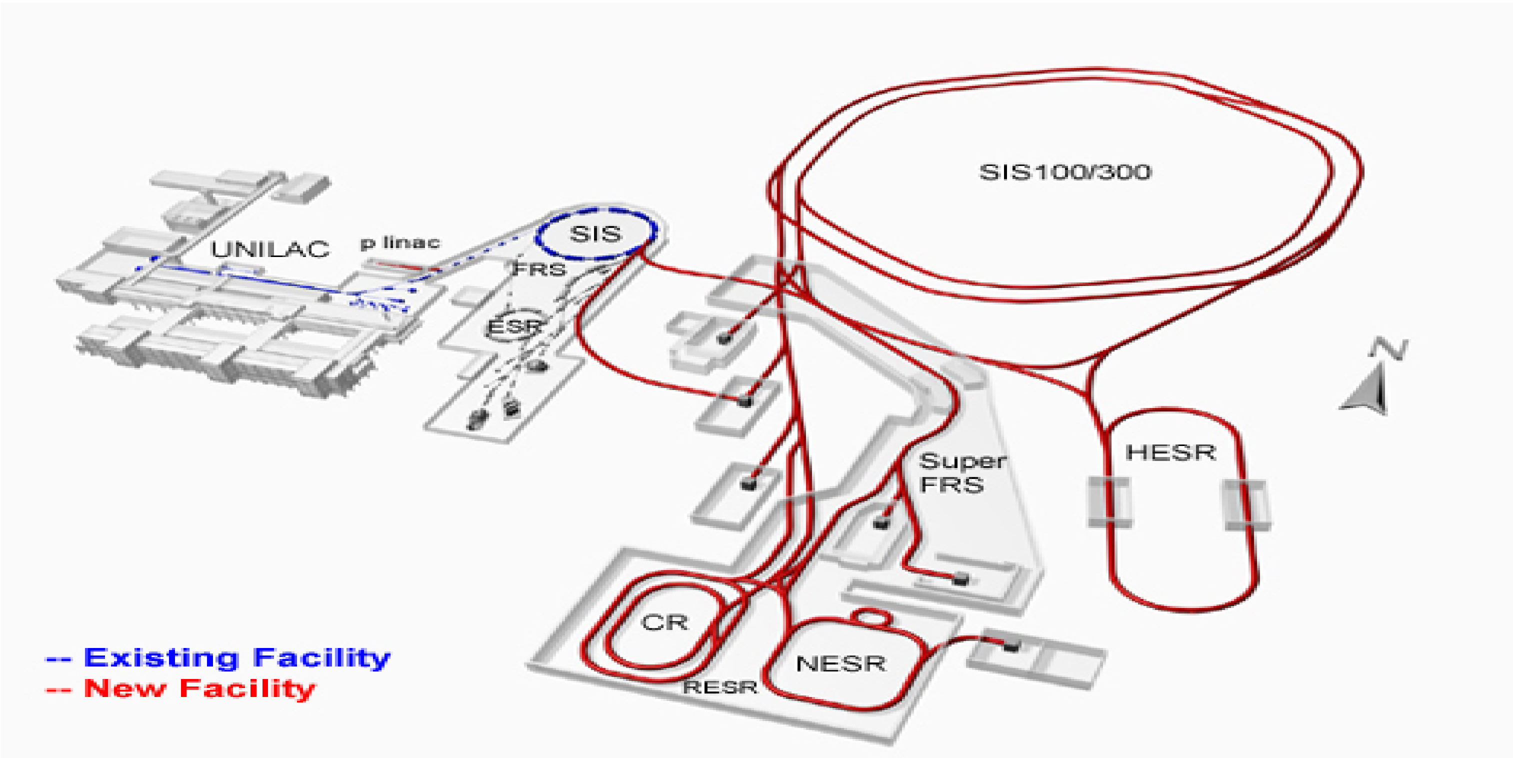
- Fully relativistic formulae for spin averaged and spin dependent (polarisation transfer) one photon exchange differential cross sections are developed for spin 1/2 fermion-fermion elastic scattering.

- These results are required by the Polarised Antiproton eXperiments (PAX) project at GSI Darmstadt.

- In particular, cross sections for polarisation transfer in polarised antiproton-electron $\bar{p}\, e\!\uparrow \longrightarrow \bar{p}\!\uparrow e$ and antiproton-proton $\bar{p}\, p\!\uparrow \longrightarrow \bar{p}\!\uparrow p$ elastic scattering are needed.

The GSI facility

The future accelerator layout



The future facility

Overview

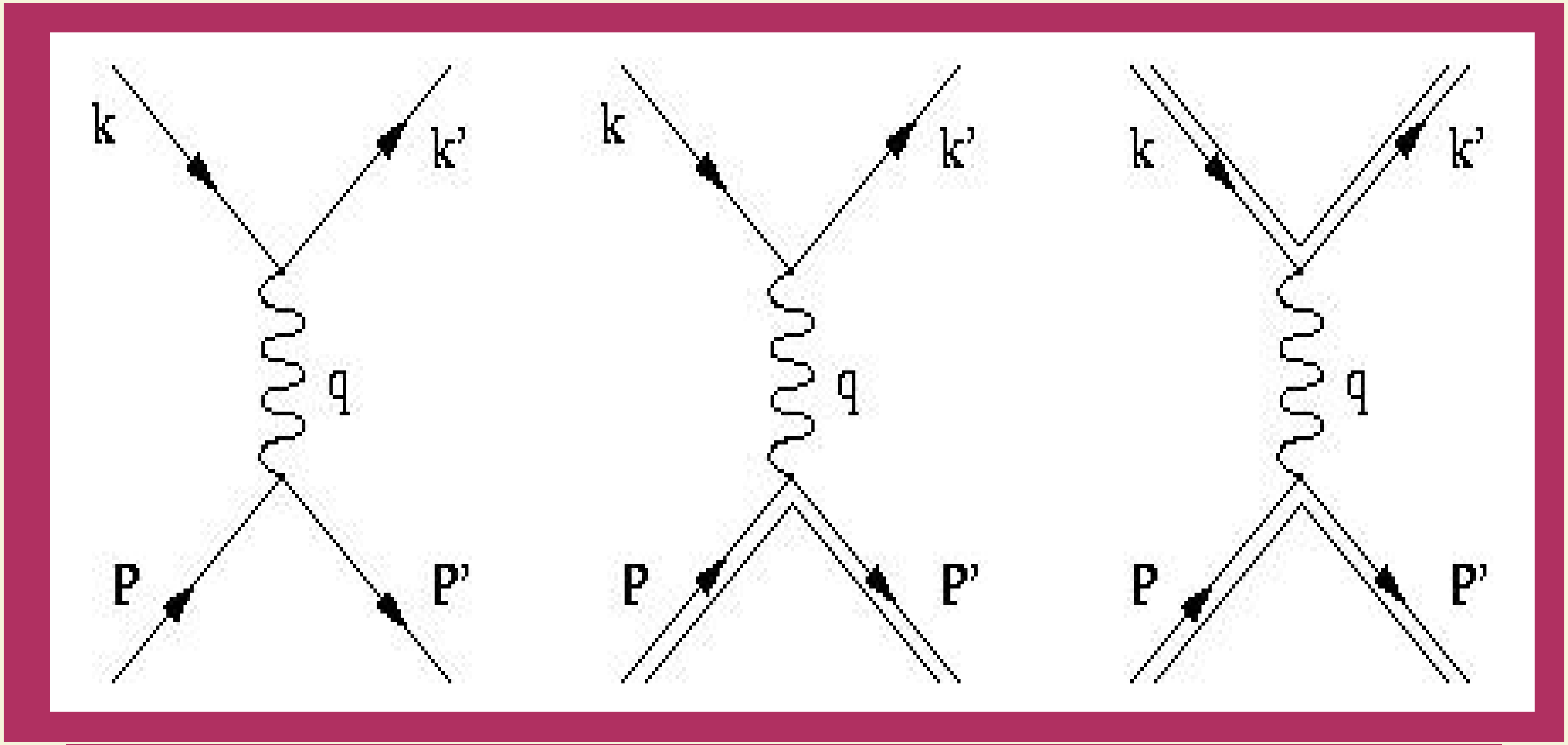


Figure 1: Feynman diagrams for the three spin averaged cases.

The basics

The differential cross section is related to the amplitude $\mathcal{M}$ by

$$s\,\frac{d\sigma}{d\Omega} = \frac{1}{(8\pi)^2}\,|\mathcal{M}|^2$$

The electron current is

$$j^\mu = e\,\bar{u}(k',\lambda')\,\gamma^\mu\,u(k,\lambda)$$

and the proton current, after Gordon decomposition is

$$J_\mu = e\,\bar{u}(P',\Lambda')\left(G_M\,\gamma_\mu - F_2\,\frac{P_\mu + P'_\mu}{2M}\right)u(P,\Lambda)$$

1 Spin averaged cases

1.1 Structureless

A standard calculation gives the differential cross-section for one photon exchange, of two non-identical spin 1/2 fermions to be

$$s\,\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{2\,t^2}\left[2\left(s - m^2 - M^2\right)^2 + 2\,s\,t + t^2\right]$$

where m and M are the masses of the particles.

The s, t and u are the Mandelstam variables, and $\alpha = e^2/4\pi$, the electromagnetic coupling constant or fine structure constant.

1.2 One particle structured

Defining electromagnetic form factors $F_1(q^2)$ and $F_2(q^2)$ such that $F_2(0) = \mu - 1$, the anomalous magnetic moment, and for convenience using $G_M = F_1 + F_2$ gives

$$s\,\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{2\,t^2}\Bigg\{G_M^2\left[2\left(s - m^2 - M^2\right)^2 + 2\,s\,t + t^2\right] - 2\,F_2\left[F_2\left(1 + \frac{t}{4M^2}\right) + 2\,F_1\right]\left[\left(s - m^2 - M^2\right)^2 + t\left(s - m^2\right)\right]\Bigg\}$$

This result equals that of the previous section in the structureless limit $F_1 \to 1$, $F_2 \to 0$ and hence $G_M \to 1$.

In the $m \to 0$ limit this is the Rosenbluth formula.

1.3 Both particles structured

Defining the electromagnetic form factors $f_1(q^2)$ and $f_2(q^2)$ for the second particle and using $g_M = f_1 + f_2$ we obtain

$$
\begin{aligned}
s\,\frac{d\sigma}{d\Omega} \;=\;& \frac{\alpha^2}{2\,t^2}\Bigg\{ g_M^2 G_M^2 \left[2\left(s - m^2 - M^2\right)^2 + 2\,s\,t + t^2 \right] \\
& - 2\,g_M^2 F_2 \left[F_2 \left(1 + \frac{t}{4M^2} \right) + 2\,F_1 \right] \left[\left(s - m^2 - M^2\right)^2 + t\,(s - m^2) \right] \\
& - 2\,G_M^2 f_2 \left[f_2 \left(1 + \frac{t}{4m^2} \right) + 2\,f_1 \right] \left[\left(s - m^2 - M^2\right)^2 + t\,(s - M^2) \right] \\
& + \frac{f_2 F_2}{2} \left[f_2 \left(1 + \frac{t}{4m^2} \right) + 2\,f_1 \right] \left[F_2 \left(1 + \frac{t}{4M^2} \right) + 2\,F_1 \right] (s-u)^2 \Bigg\}\,.
\end{aligned}
$$

Again this result equals that of the previous section in the one particle structured limit $f_1 \to 1$, $f_2 \to 0$ and hence $g_M \to 1$.

1.4 Antiproton proton case

For antiproton-proton collisions, the electromagnetic form factors and masses of the proton and antiproton are the same, i.e. $f_1 = F_1$ and $f_2 = F_2$ so $g_M = G_M$; and $m = M$. Here we neglect the s-channel one photon contribution in favour of the t-channel term which dominates in the low momentum transfer (small t) region of interest, and also dominates at high energies. This gives

$$\begin{aligned} s\,\frac{d\sigma}{d\Omega} \;=\; \frac{\alpha^2}{2\,t^2}\Bigg\{ & G_M^4\left[2\left(s-2M^2\right)^2 + 2\,s\,t + t^2\right] \\ & - 4\,G_M^2 F_2\left[F_2\left(1+\frac{t}{4M^2}\right) + 2\,F_1\right]\left[\left(s-2M^2\right)^2 + t\left(s-M^2\right)\right] \\ & + \frac{F_2^2}{2}\left[F_2\left(1+\frac{t}{4M^2}\right) + 2\,F_1\right]^2 (s-u)^2 \Bigg\} \end{aligned}$$

This agrees with an expression formed from known fermion fermion helicity amplitudes (N. H. Buttimore *et al.*, Phys. Rev. D **18** (1978) 694), and with a cross section formula for proton proton scattering (M. M. Block Phys. Rev. D **54** (1996) 4337). In this case $s + t + u = 4M^2$ and thus $s - u = 2\,s + t - 4M^2$.

This result is important in the momentum transfer region $|t| < |t_c|$ for antiproton proton collisions with total cross section $\sigma_{\rm tot}$, defined by

$$t_c = -\frac{8\,\pi\,\alpha}{\beta_{\rm lab}\,\sigma_{\rm tot}} \approx -\,0.001\,(\text{GeV}/c)^2$$

where the electromagnetic interaction dominates the hadronic interaction. Here the laboratory velocity is $\beta_{\rm lab} = \sqrt{s(s-4M^2)}/(s-2M^2)$.

2 Spin dependent cases

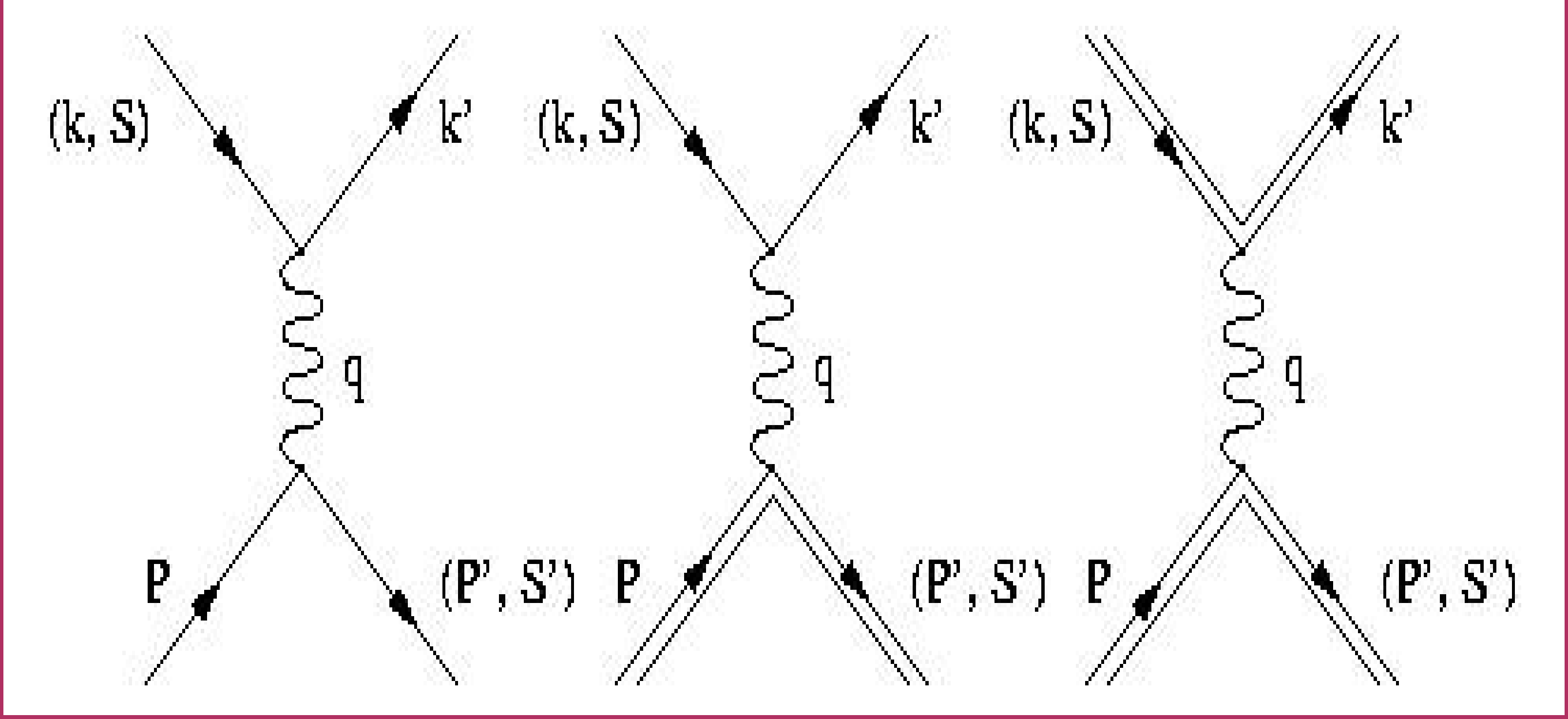


Figure: Feynman diagrams for the three spin dependent cases.

The basics

- Suppose the initial electron (or proton) to have a spin four vector S_μ and the final scattered antiproton to have a spin four vector S'_μ.

- We are most interested in the polarisation transfer K_{j00i}, i.e. $\bar{p}\,p\!\uparrow\,\longrightarrow \bar{p}\!\uparrow p$.

$$\bar{p}(P) + p(k,S) \longrightarrow \bar{p}(P',S') + p(k')$$

- Consider just the spin dependent terms, and use the notation $A \cdot B = A_\mu B^\mu$.

- Define the momentum transfer to the photon $q = k' - k = P - P'$ by conservation of 4-momentum, and use $q^2 = t$.

2.1 Structureless

The cross section for polarisation transfer K_{j00i}, from initial electron to final antiproton (assumed structureless here) is

$$s\,\frac{d\sigma}{d\Omega}\,K_{j00i} = -\left(\frac{2\,\alpha^2}{t}\right) mM\left[S\cdot S' - \frac{S\cdot q\,S'\cdot q}{t}\right].$$

2.2 One particle structured

Using the proton electromagnetic form factors as described earlier, and also $S \cdot k = 0$ and $S' \cdot P' = 0$ from the general theory of spin polarisation.

$$
\begin{aligned}
s \frac{d\sigma}{d\Omega} K_{j00i} \quad = \quad & - \left(\frac{2\,\alpha^2}{t} \right) m M G_M \left\{ F_1 \left[S \cdot S' - \frac{S \cdot q\, S' \cdot q}{t} \right] \right. \\
& \left. + \frac{F_2}{4M^2} \left[t\, S \cdot S' + 2\, S \cdot P'\, S' \cdot q \right] \right\}
\end{aligned}
$$

This result equals that of the previous section in the limit $F_1 \to 1$, $F_2 \to 0$ and hence $G_M \to 1$.

This represents a relativistic generalisation of equation (3) of Horowitz & Meyer, PRL **72** (1994) 3981.

2.3 Both particles structured

Using the electromagnetic form factors $f_1(q^2)$ and $f_2(q^2)$ of the second particle as earlier we obtain

$$\left(\frac{-t}{2\,\alpha^2}\right) s\,\frac{d\sigma}{d\Omega}\,K_{j00i} =$$

$$= mMg_M^2 G_M \left\{F_1\left[S\cdot S' - \frac{S\cdot q\,S'\cdot q}{t}\right] + \frac{F_2}{4M^2}\left[t\,S\cdot S' + 2\,S\cdot P'\,S'\cdot q\right]\right\}$$

$$+\frac{f_2}{m}\,MG_M^2 g_M\left\{S\cdot S'\left[t\,\left(t-4m^2\right)\right] + 4\,m^2 S\cdot q\,S'\cdot q + 2\,t\,S\cdot q\,S'\cdot k\right\}$$

$$+\frac{f_2}{m}\,\frac{F_2}{M}\,G_M g_M k_\tau k'_\alpha P^\lambda P'^\rho S_\beta S'^\sigma\left[(s-u)\epsilon^{\mu\alpha\beta\tau}\epsilon_{\mu\sigma\rho\lambda}\right.$$

$$\left. -\epsilon^{\mu\alpha\beta\tau}\epsilon_{\nu\sigma\rho\lambda}\left(k^\nu + k'^\nu\right)\left(P_\mu + P'_\mu\right)\right]\,.$$

Again this equals the previous formulae in the appropriate limits.

2.4 Antiproton proton case

Again the proton and antiproton electromagnetic form factors and masses are the same, and the s-channel term is neglected in favour of the t-channel contribution in the low momentum transfer region of interest. Thus

$$\left(\frac{-t}{2\,\alpha^2}\right) s\,\frac{d\sigma}{d\Omega}\,K_{j00i} =$$

$$= M^2 G_M^3 \left\{ F_1 \left[S\cdot S' - \frac{S\cdot q\, S'\cdot q}{t} \right] + \frac{F_2}{4M^2} \left[t\, S\cdot S' + 2\, S\cdot P'\, S'\cdot q \right] \right\}$$

$$+ F_2\, G_M^3 \left\{ S\cdot S' \left[t\, \left(t - 4M^2 \right) \right] + 4\, M^2 S\cdot q\, S'\cdot q + 2\, t\, S\cdot q\, S'\cdot k \right\}$$

$$+ \frac{F_2^2}{M^2} G_M^2 k_\tau k'_\alpha P^\lambda P'^\rho S_\beta S'^\sigma \left\{ \epsilon^{\mu\alpha\beta\tau} \left[\epsilon_{\mu\sigma\rho\lambda} (s-u) - \epsilon_{\nu\sigma\rho\lambda} \left(k^\nu + k'^\nu \right) \left(P_\mu + P'_\mu \right) \right] \right\}$$

and again this result is important in the momentum transfer region $|t| < |t_c|$ defined earlier, where the electromagnetic interaction dominates the hadronic interaction.

Conclusions

- Fully relativistic differential cross section formulae have been derived for polarisation transfer in spin $1/2$ fermion-fermion elastic scattering, due to one photon exchange.

- They can be applied to proton-electron and antiproton-electron scattering, and to near forward (small t) antiproton-proton scattering.

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