Quantum field theory of fundamental particle interactions

Donie O'Brien

School of Mathematics, Trinity College Dublin. Email: donie@maths.tcd.ie

Supervisor: Nigel Buttimore

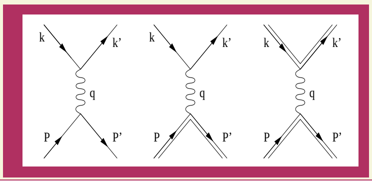

Figure 1: Feynman diagrams for the three spin averaged cases.

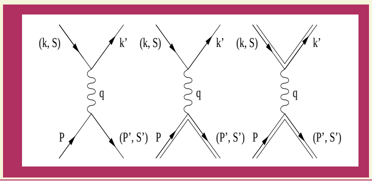

Figure 2: Feynman diagrams for the three spin dependent cases.

Abstract

Relativistic formulae for spin averaged and spin dependent one photon exchange differential cross sections are developed for spin 1/2 fermion-fermion elastic scattering [1]. These results are required by the Polarized Antiproton eXperiments (PAX) project at GSI Darmstadt. In particular, cross sections for polarization transfer in polarized antiproton-electron $\bar{p}\, e\!\uparrow\ \longrightarrow \bar{p}\!\uparrow e$ and antiproton-proton $\bar{p}\, p\!\uparrow\ \longrightarrow \bar{p}\!\uparrow p$ elastic collisions are presented.

1. Introduction

Antiprotons were discovered fifty years ago. We investigate the results of elastic scattering of antiprotons off electrons and protons, due to single photon exchange, first unpolarized in sections 2 and 3. Fully relativistic polarization transfer differential cross sections are derived for elastic antiproton-electron and antiproton-proton Coulomb scattering in section 4. In section 5 our results are summarized.

Emphasis is put on the spin transfer differential cross section for elastic antiproton-proton scattering, as it has been suggested in a recent paper that electrons are not effective in transferring polarization to antiprotons in the kinematic region of the PAX programme [2].

2. Cross sections

The differential cross section is related to the helicity amplitudes $\mathcal{M}(\Lambda'\lambda';\Lambda\,\lambda)$ by

$$s\,\frac{d\sigma}{d\Omega} = \frac{1}{(8\pi)^2}\sum_{\lambda\lambda'\Lambda\Lambda'}\frac{1}{(2\lambda+1)(2\Lambda+1)}\,|\mathcal{M}(\Lambda'\lambda';\Lambda\,\lambda)|^2$$

where λ,Λ and λ',Λ' are the helicities of the initial and final particles respectively, and the s and t are Mandelstam variables.

3. Spin averaged cases

Structureless

A standard calculation gives the differential cross section for one photon exchange, of two non-identical spin 1/2 point particles to be

$$s\,\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{2\,t^2}\left[\,2\left(s-m^2-M^2\right)^2+2\,s\,t+t^2\right]$$

where m and M are the masses of the particles, and $\alpha = e^2/4\pi$.

One particle structured

Consider one particle to have structure determined by the electromagnetic form factors $F_1(q^2)$ and $F_2(q^2)$ normalized to $F_1(0) = 1$ and $F_2(0) = \mu - 1$, the anomalous magnetic moment. Using $t = q^2$ and $G_M = F_1 + F_2$,

$$s\,\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{2\,t^2}\left\{G_M^2\left[\,2\left(s-m^2-M^2\right)^2+2\,s\,t+t^2\right]\right.$$
$$\left.-\,2\,F_2\left[F_2\left(1+\frac{t}{4M^2}\right)+2\,F_1\right]\left[\left(s-m^2-M^2\right)^2+t\left(s-m^2\right)\right]\right\}$$

which agrees with the previous result in the structureless limit $F_1 \to 1$, $F_2 \to 0$ and hence $G_M \to 1$. In the $m \to 0$ limit this is the famous Rosenbluth formula [3].

4. Spin dependent cases

Suppose the initial target electron (or proton) to have a spin four vector S_μ and the final scattered antiproton to have a spin four vector S'_μ.
We are most interested in the polarization transfer K_{j00i}, i.e. $\bar{p}\, p\!\uparrow\ \longrightarrow \bar{p}\!\uparrow p$.

Structureless

We find the one photon exchange cross section for polarization transfer from initial electron to final antiproton (assumed structureless here) to be

$$s\,\frac{d\sigma}{d\Omega}\,K_{j00i} = -\left(\frac{2\,\alpha^2}{t}\right)mM\left[\,S\cdot S' - \frac{S\cdot q\,S'\cdot q}{t}\right].$$

One particle structured

Using the electromagnetic form factors as described earlier, and also $S\cdot k = 0$ and $S'\cdot P' = 0$ from the theory of spin polarization,

$$\begin{aligned} s\,\frac{d\sigma}{d\Omega}\,K_{j00i} &= -\left(\frac{2\,\alpha^2}{t}\right)mMG_M\left\{F_1\left[\,S\cdot S' - \frac{S\cdot q\,S'\cdot q}{t}\right]\right.\\ &\left.\quad+\frac{F_2}{4M^2}\left[t\,S\cdot S' + 2\,S\cdot P'\,S'\cdot q\right]\right\}\end{aligned}$$

which equals the previous result in the limit $F_1 \to 1$, $F_2 \to 0$ and $G_M \to 1$. This represents a relativistic generalization of equation (3) of reference [4].

Both particles structured

We have also derived results for both particles structured, in the spin averaged and spin dependent cases. Specifically results for polarization transfer in antiproton-proton interactions have been obtained [1], which are required by the PAX collaboration.

5. Summary

The spin averaged and polarization transfer elastic differential cross sections due to one photon exchange presented here may assist in understanding the spin filtering process that seeks to increasingly polarize antiprotons in a storage ring, in order to measure a number of significant properties of the proton. In particular, the transversity distribution of the valence quarks in addition to the analytic properties of the time-like electromagnetic form factors would be measured for the first time.

These results can be applied to proton-electron and antiproton-electron scattering, and to near forward (small t) antiproton-proton scattering where the Coulomb interaction is important.

Acknowledgments

I would like to thank 'The Embark Initiative', for a postgraduate research scholarship, and Dr. Nigel Buttimore for helpful guidance, who is grateful for an International Collaboration Programme award.

References

[1] D. S. O'Brien and N. H. Buttimore, Czech. J. Phys. **56**, C265 (2006).

[2] A. I. Milstein and V. M. Strakhovenko, Phys. Rev. E **72**, 066503 (2005).

[3] M. N. Rosenbluth, Phys. Rev. **79**, 615 (1950).

[4] C. J. Horowitz and H. O. Meyer, Phys. Rev. Lett. **72**, 3981 (1994).

Polarized Antiproton EXperiments

Supported by