Polarization buildup by spin filtering in storage rings

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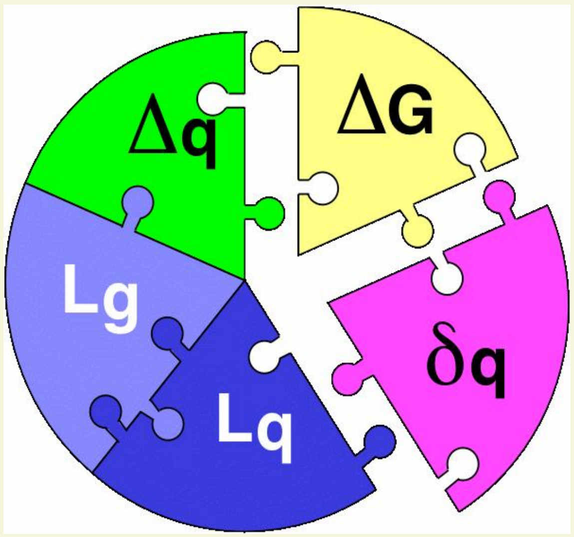
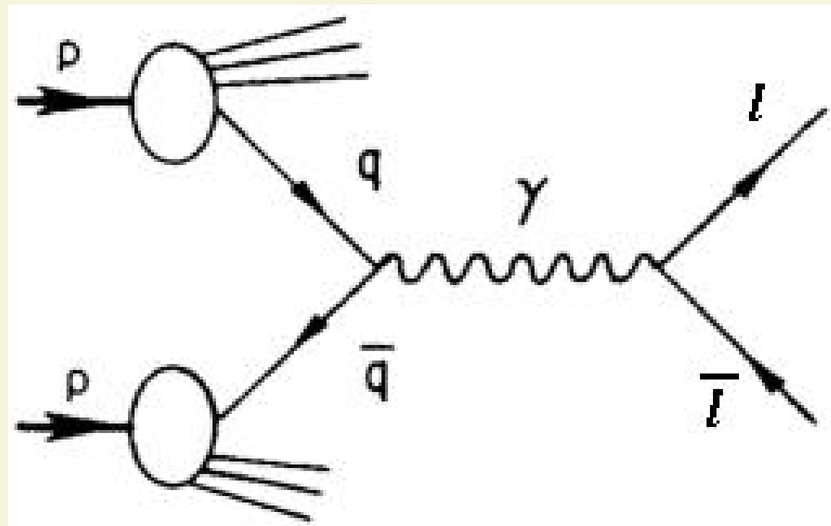
Introduction

- The $\mathcal{PAX}$ project at GSI Darmstadt plans to polarize an antiproton beam by repeated interaction with a hydrogen target in a storage ring.
- Many of the beam particles are required to remain within the ring after interaction with the target so small scattering angles are important. Hence we concentrate on low momentum transfer (small $-t = Q^2$).
- Electromagnetic effects dominate the hadronic effects in this low $-t$ region of interest. Thus we calculate all Electromagnetic Helicity amplitudes and Spin Observables for elastic $\bar{p}\,p$ and $\bar{p}\,e$ scattering, to first order in QED.
- A beam of polarized electrons with energy sufficient to provide scattering of antiprotons beyond ring acceptance may polarize an antiproton beam by spin filtering.
- The spin observables are then used to estimate the rate of buildup of polarization of an antiproton beam.

1 Why do we want a polarized antiproton beam





In order to access the transversity distribution function the double spin asymmetry A_{TT} in the Drell-Yan production of lepton pairs, must be measured; thus both initial particles in a reaction must be transversely polarized. It could in future been done for $p^{\uparrow}p^{\uparrow}$ scattering at RHIC, but this asymmetry is expected to be small from theory, as explained below.

$$A_{\mathrm{TT}}^{p\,p} = \frac{d\Delta\hat{\sigma}}{d\hat{\sigma}} \frac{\sum_q e_q^2 \left[h_{1\,q}^{\,p}\left(x_1, M^2\right) h_{1\,\bar{q}}^{\,p}\left(x_2, M^2\right) + h_{1\,\bar{q}}^{\,p}\left(x_1, M^2\right) h_{1\,q}^{\,p}\left(x_2, M^2\right) \right]}{\sum_q e_q^2 \left[q\left(x_1, M^2\right)\ \bar{q}\left(x_2, M^2\right) + \bar{q}\left(x_1, M^2\right)\ q\left(x_2, M^2\right) \right]}$$

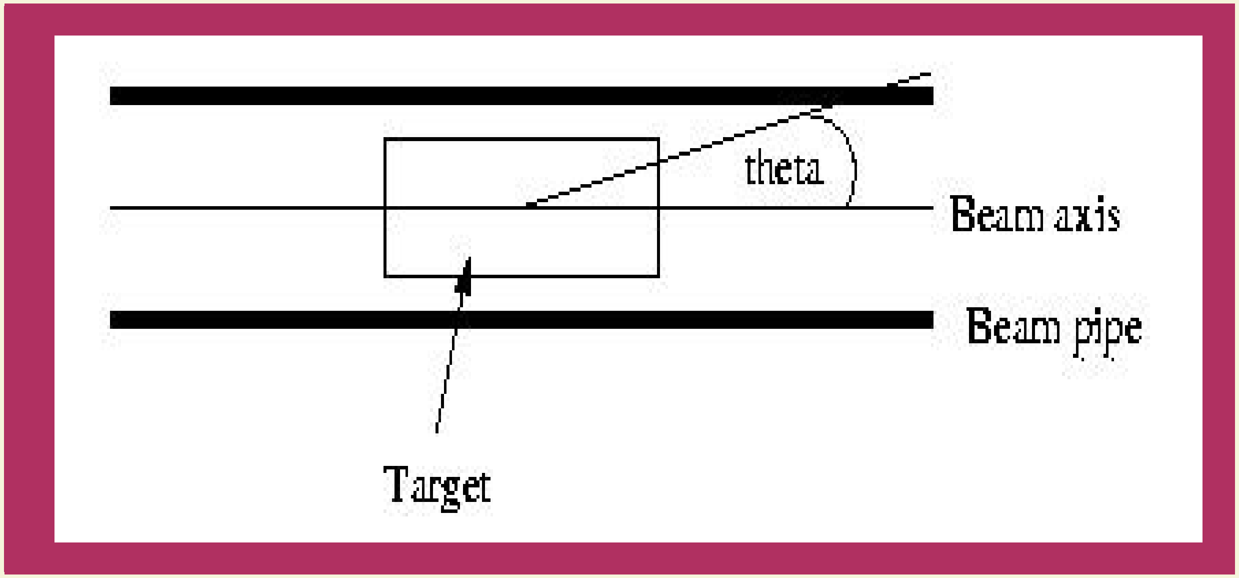
$$\approx \quad \frac{d\Delta\hat{\sigma}}{d\hat{\sigma}} \frac{h_{1\,u}^{\,p}\left(x_1, M^2\right)\ \ h_{1\,\bar{u}}^{\,p}\left(x_2, M^2\right)}{u\left(x_1, M^2\right)\ \bar{u}\left(x_2, M^2\right)}$$

$$A_{\mathrm{TT}}^{p\,\bar{p}} = \frac{d\Delta\hat{\sigma}}{d\hat{\sigma}} \frac{\sum_q e_q^2 \left[h_{1\,q}^{\,p}\left(x_1, M^2\right) h_{1\,\bar{q}}^{\,\bar{p}}\left(x_2, M^2\right) + h_{1\,\bar{q}}^{\,p}\left(x_1, M^2\right) h_{1\,q}^{\,\bar{p}}\left(x_2, M^2\right) \right]}{\sum_q e_q^2 \left[q\left(x_1, M^2\right)\ \bar{q}\left(x_2, M^2\right) + \bar{q}\left(x_1, M^2\right)\ q\left(x_2, M^2\right) \right]}$$

$$\approx \quad \frac{d\Delta\hat{\sigma}}{d\hat{\sigma}} \frac{h_{1\,u}^{\,p}\left(x_1, M^2\right)\ \ h_{1\,\bar{u}}^{\,\bar{p}}\left(x_2, M^2\right)}{u\left(x_1, M^2\right)\ \bar{u}\left(x_2, M^2\right)},$$

the latter of which is much larger since there are more antiquarks in antiprotons making $h_{1\,\bar{q}}^{\,\bar{p}}\left(x_2, Q^2\right) \gg h_{1\,\bar{q}}^{\,p}\left(x_2, Q^2\right)$. Thus $A_{\mathrm{TT}}^{p\,\bar{p}}$, which can only be measured using a polarized antiproton beam, is expected to be much bigger than $A_{\mathrm{TT}}^{p\,p}$.

The Spin Filtering Idea



The target is initially polarized and the beam is initially unpolarized, we seek to model the buildup of polarization of the antiproton beam.

$$\sigma_{\mathrm{tot}}^{\bar{p}\,p}(\uparrow\uparrow) \neq \sigma_{\mathrm{tot}}^{\bar{p}\,p}(\downarrow\uparrow)$$

For $p\,p$ spin filtering CM energies should be less than the pion production threshold, to avoid beam losses due to inelastic collisions.

2 Normalization

We investigate the general two particle elastic process with spin

$$A\,(\,p_1,\,S_1\,) + B\,(\,p_2,\,S_2\,) \longrightarrow A\,(\,p_3,\,S_3\,) + B\,(\,p_4,\,S_4\,)$$

where it is assumed that the beam particles (A) of mass M are antiprotons and the target particles (B) of mass m are electrons or protons .

The target is initially polarized and the beam is initially unpolarized, we seek to model the buildup of polarization of the antiproton beam. We look at the following elastic processes

$$\begin{array}{ccc} \bar{p}\ e^{\uparrow} & \longrightarrow & \bar{p}^{\uparrow}\ e \\ \bar{p}\ p^{\uparrow} & \longrightarrow & \bar{p}^{\uparrow}\ p \end{array}$$

3.1 The Generic Calculation

The generic equation for polarization effects in elastic spin 1/2 - spin 1/2 scattering to first order in QED is

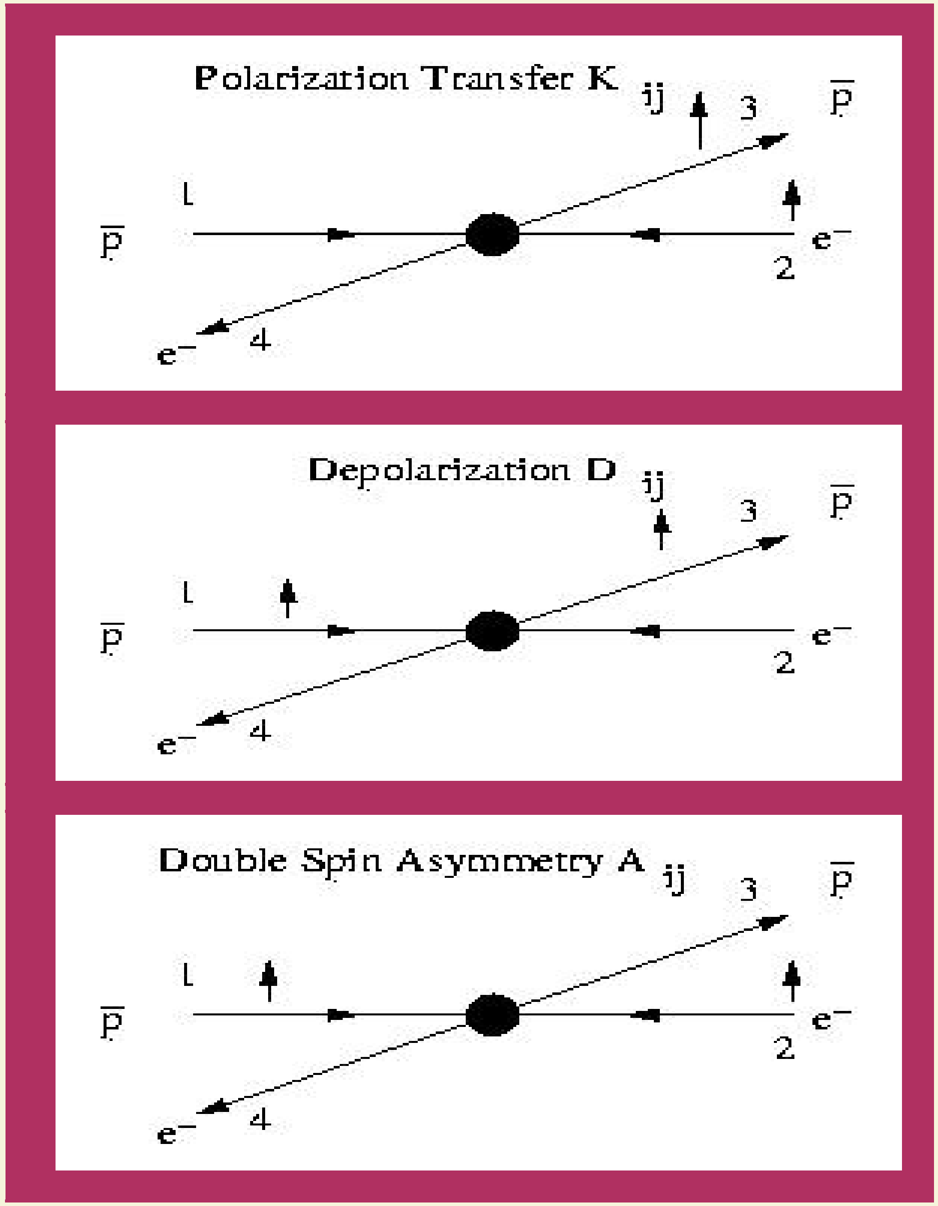
$$16\left(\frac{q}{e}\right)^4 |\mathcal{M}|^2 =$$

$$\mathsf{Tr}\left[\left(\not{p}_4 + m\right)(1+\gamma_5\not{S}_4)\,(g_M\gamma^\nu + fr^\nu)\left(\not{p}_2 + m\right)(1+\gamma_5\not{S}_2)\,(g_M\gamma^\mu + fr^\mu)\right]\times$$

$$\mathsf{Tr}\left[\left(\not{p}_1 + M\right)(1+\gamma_5\not{S}_1)\,(G_M\gamma_\mu + FR_\mu)\left(\not{p}_3 + M\right)(1+\gamma_5\not{S}_3)(G_M\gamma_\nu + FR_\nu)\right]$$

where the electromagnetic form factors $G_M = F_1 + F_2$, $g_M = f_1 + f_2$, $F = -F_2/2M$ and $f = -f_2/2m$; also $R^\mu = p_1^\mu + p_3^\mu$, $r^\mu = p_2^\mu + p_4^\mu$. This generic equation can thus be used to calculate all helicity amplitudes and spin observables etc. by substituting specific values for the spin (S_i) and momenta (p_i) vectors. The result has been obtained for this equation with the traces computed and contracted, using *Mathematica*.

Centre-of-Mass Momenta vectors	
$p_1 = (E_1, 0, 0, k)$	$p_3 = (E_1, k\sin\theta, 0, k\cos\theta)$
$p_2 = (E_2, 0, 0, -k)$	$p_4 = (E_2, -k\sin\theta, 0, -k\cos\theta)$
Centre-of-Mass Transverse spin vectors	
$S_1^X = (0, 1, 0, 0)$	$S_3^X = (0, \cos\theta, 0, -\sin\theta)$
$S_2^X = (0, 1, 0, 0)$	$S_4^X = (0, -\cos\theta, 0, \sin\theta)$
Centre-of-Mass Normal spin vectors	
$S_1^Y = (0, 0, 1, 0)$	$S_3^Y = (0, 0, 1, 0)$
$S_2^Y = (0, 0, 1, 0)$	$S_4^Y = (0, 0, 1, 0)$
Centre-of-Mass Longitudinal spin vectors	
$S_1^Z = \frac{1}{M}(k, 0, 0, E_1)$	$S_3^Z = \frac{1}{M}(k, E_1\sin\theta, 0, E_1\cos\theta)$
$S_2^Z = \frac{1}{m}(k, 0, 0, -E_2)$	$S_4^Z = \frac{1}{m}(k, -E_2\sin\theta, 0, -E_2\cos\theta)$


Polarization Transfer K_{ij}
$\bar{p}$
e^-
L
1
2
3
4
Depolarization D_{ij}
$\bar{p}$
e^-
L
2
3
4
Double Spin Asymmetry A_{ij}
$\bar{p}$
e^-
L
2
3
4

4 Spin Observables

- All the electromagnetic spin observables of a reaction (polarization transfer K_{ij}, depolarization $(1 - D_{ij})$ and asymmetries A_{ij} where $i, j, k \in \{\mathrm{X}, \mathrm{Y}, \mathrm{Z}\}$) can now be obtained by direct computation. See D.O'B. and N. H. Buttimore hep-ph/0609233 for complete results.

- For electromagnetic interactions to first order the double spin asymmetries equal the polarization transfer observables ($A_{ij} = K_{ij}$) and all the single and triple spin asymmetries are zero ($A_i = A_{ijk} = 0$).

- Spin filtering requires evaluation of the angular integration of the product of the observables $A_{ii} = K_{ii}$ and $(1 - D_{ii})$ with $d\,\sigma/d\,\Omega$. Azimuthal averaging indicates that the observables with single X (i.e. K_{XZ}, K_{ZX}, D_{XZ} and D_{ZX}) do not contribute to spin filtering. The quantities $(K_{\mathrm{XX}} + K_{\mathrm{YY}})/2$, $(D_{\mathrm{XX}} + D_{\mathrm{YY}})/2$, K_{ZZ} and D_{ZZ} play the important role, we now present results for these.

4.1 Antiproton-proton scattering

To look at the case of antiproton-proton scattering set the form factors and masses of each particle equal ($f_1 \to F_1$, $f_2 \to F_2$ and $m \to M$) in the generic equation. We obtain the results to leading order in small t:

$$\frac{K_{\mathrm{XX}} + K_{\mathrm{YY}}}{2} \frac{d\,\sigma}{d\,\Omega} \approx \frac{\alpha^2\ M^{\,2}\ \mu^{\,2}}{s\ t}$$

$$\frac{(1 - D_{\mathrm{XX}}) + (1 - D_{\mathrm{YY}})}{2} \frac{d\,\sigma}{d\,\Omega} \approx \frac{-\ \alpha^2 \left(k^2 + M^{\,2}\right)}{k^2\ M^{\,2}\ s\ t} \left[M^{\,2} - 2\,k^2\,(\mu - 1)\right]^{\,2}$$

$$K_{\mathrm{ZZ}} \frac{d\,\sigma}{d\,\Omega} \approx \frac{-\,2\ \alpha^2\ \mu^2}{s\ t} \left(2\,k^2 + M^{\,2}\right)$$

$$(1 - D_{\mathrm{ZZ}}) \frac{d\,\sigma}{d\,\Omega} \approx \frac{-\,2\,\alpha^2 \left(k^2 + M^{\,2}\right)}{k^2\ M^{\,2}\ s\ t} \left[M^{\,2} - 2\,k^2\,(\mu - 1)\right]^{\,2}$$

4.2 Antiproton-electron scattering

To look at the case of antiproton-electron scattering set the form factors of the second particle to be structureless ($f_1 \to 1$ and $f_2 \to 0$) in the generic equation. We obtain the results to leading order in small t:

$$\frac{K_{\mathrm{XX}} + K_{\mathrm{YY}}}{2} \frac{d\,\sigma}{d\,\Omega} \approx \frac{\alpha^2\ m\ M\ \mu}{s\ t}$$

$$\frac{(1 - D_{\mathrm{XX}}) + (1 - D_{\mathrm{YY}})}{2} \frac{d\,\sigma}{d\,\Omega} \approx \frac{-m^2\,\alpha^2\,\left(s - m^2 + M^2\right)^2}{4\ k^2\ s^2\ t}$$

$$K_{\mathrm{ZZ}} \frac{d\,\sigma}{d\,\Omega} \approx \frac{-\,\alpha^2\ \mu}{s\ t}\left(s - m^2 - M^2\right)$$

$$(1 - D_{\mathrm{ZZ}}) \frac{d\,\sigma}{d\,\Omega} \approx \frac{-M^2\,\alpha^2\,\left(s + m^2 - M^2\right)^2}{2\ k^2\ s^2\ t}$$

Antiproton-proton spin observables

$$\frac{d\,\sigma}{d\,\Omega}\,K_{\mathrm{XX}} = \frac{\alpha^2\,G_M^{\,2}}{8\,s\,k^{\,2}\,M^2}\left\{4\,M^{\,4}F_1^{\,2}-8\,k^2M^{\,2}F_1\,F_2+\left[4\,k^4+\left(k^2+\frac{t}{4}\right)\,s\right]\,F_2^{\,2}\right\}$$

$$\frac{d\,\sigma}{d\,\Omega}\,K_{\mathrm{YY}} = \left(\frac{2\,\alpha^2}{s\,t}\right)\,M^{\,2}\,G_E^{\,2}\,G_M^{\,2}$$

$$\frac{d\,\sigma}{d\,\Omega}\,K_{ZZ} = \frac{-\,\alpha^2\,G_M^{\,2}}{8\,k^2\,s\,\,t}\left[\,s\left(4\,k^2+t\right)\,F_1^{\,2}+\left(\,4\,k^2\,F_1-t\,F_2\right)^{\,2}\right]$$

$$\frac{d\,\sigma}{d\,\Omega}\,K_{XZ} = \frac{d\,\sigma}{d\,\Omega}\,K_{ZX} = \frac{\alpha^2\,G_M^{\,2}\sqrt{s}}{2\,M\,t}\sqrt{\frac{-\,t\,\left(\,4\,k^2+t\,\right)}{k^{\,4}}}\left(\frac{M^2\,F_1^{\,2}}{2}-k^2\,F_1\,F_2+\frac{t\,F_2^{\,2}}{8}\right)$$

$$\frac{d\,\sigma}{d\,\Omega}\,\left(\,1-D_{\mathrm{XX}}\,\right) \approx \frac{-\,2\,\alpha^2\,F_1^{\,2}}{k^{\,2}\,M^{\,2}\,s\,t}\left(\,k^{\,2}+M^{\,2}\right)\left(\,M^{\,2}\,F_1-2\,k^2\,F_2\right)^{\,2}$$

$$\frac{d\,\sigma}{d\,\Omega}\,\left(\,1-D_{\mathrm{YY}}\,\right) = \frac{\alpha^2}{2\,\,s}\,G_M^{\,4}\,, \qquad \text{complete to all orders in } t$$

$$\frac{d\,\sigma}{d\,\Omega}\,\left(\,1-D_{\mathrm{ZZ}}\right) \approx \frac{-\,2\,\alpha^2\,F_1^{\,2}}{k^{\,2}\,M^{\,2}\,s\,t}\left(\,k^{\,2}+M^{\,2}\right)\left(\,M^{\,2}\,F_1-2\,k^2\,F_2\right)^{\,2}$$

$$\frac{d\,\sigma}{d\,\Omega}\,\left(\,1-D_{\mathrm{XZ}}\,\right) \approx \frac{d\,\sigma}{d\,\Omega}\,\left(\,1-D_{\mathrm{ZX}}\,\right) \approx \frac{d\,\sigma}{d\,\Omega} \approx \frac{4\,\,\alpha^2\,\,F_1^{\,4}}{s\,\,t^{\,2}}\left(\,2\,\,k^{\,2}+M^{\,2}\right)^{\,2}$$

Antiproton-electron spin observables

$$\frac{d\sigma}{d\Omega} K_{XX} = \alpha^2 \frac{m\, G_M}{2\, k^2\, M\, s} \left(M^2 F_1 - k^2 F_2 \right)$$

$$\frac{d\sigma}{d\Omega} K_{YY} = \left(\frac{2\,\alpha^2}{s\ t} \right) m\, M\, G_E\, G_M$$

$$\frac{d\sigma}{d\Omega} K_{ZZ} = \frac{-\alpha^2\, G_M}{8\, k^2\, s^2\, t} \left\{ \left[s^2 - \left(M^2 - m^2 \right)^2 \right] \left(4\, k^2 + t \right) F_1 + 4\, k^2\, s \left(4\, k^2 F_1 - t\, F_2 \right) \right\}$$

$$\frac{d\sigma}{d\Omega} K_{XZ} = \frac{\alpha^2\, m\, F_1\, G_M}{4\, s^{3/2}\, t} \sqrt{\frac{-t\left(4\, k^2 + t\right)}{k^4}} \left(s - m^2 + M^2 \right)$$

$$\frac{d\sigma}{d\Omega} K_{ZX} = \frac{\alpha^2\, G_M}{4\, M\, s^{3/2}\, t} \sqrt{\frac{-t\left(4\, k^2 + t\right)}{k^4}} \left[M^2 \left(s + m^2 - M^2 \right) F_1 - 2\, k^2 s\, F_2 \right]$$

$$\frac{d\sigma}{d\Omega} \left(1 - D_{\mathrm{XX}} \right) \approx \frac{-m^2\, \alpha^2\, F_1^2}{2\ k^2\ s^2\ t} \left(s - m^2 + M^2 \right)^2$$

$$\frac{d\sigma}{d\Omega} \left(1 - D_{\mathrm{YY}} \right) = \frac{\alpha^2}{2\ s} G_M^2 , \qquad \text{complete to all orders in } t$$

$$\frac{d\sigma}{d\Omega} \left(1 - D_{\mathrm{ZZ}} \right) \approx \frac{-M^2\, \alpha^2\, F_1^2}{2\ k^2\ s^2\ t} \left(s + m^2 - M^2 \right)^2$$

$$\frac{d\sigma}{d\Omega} \left(1 - D_{\mathrm{XZ}} \right) \approx \frac{d\sigma}{d\Omega} \left(1 - D_{\mathrm{ZX}} \right) \approx \frac{d\sigma}{d\Omega} \approx \frac{4\ \alpha^2\ F_1^2}{s\ t^2} \left(s\ k^2 + m^2\ M^2 \right)$$

5 The $\mathcal{PAX}$ Project

- There has been much recent debate as to whether electrons in the hydrogen target will transfer polarization to the antiproton beam.

- We're investigating if a beam of polarized electrons with sufficiently high density could be used to polarize an antiproton beam.

5.1 Polarization buildup

When circulating at frequency ν through a polarized target of areal density n and polarization P_e ,

$$\frac{d}{dt}\begin{bmatrix} N \\ J \end{bmatrix} = -n\,\nu \begin{bmatrix} I_{\rm out} & P_e\,A_{\rm out} \\ P_e\,A_{\rm all} - P_e\,K_{\rm in} & I_{\rm all} - D_{\rm in} \end{bmatrix} \begin{bmatrix} N \\ J \end{bmatrix}$$

describes the rate of change of the number of beam particles N and their total spin J. (See N. N. Nikolaev and F. F. Pavlov hep-ph/0601184)

These coupled differential equations involve angular integration of the spin observables presented earlier.

Transverse polarization requires	Longitudinal polarization requires
$I_{\rm out} = 2\pi \int_{\theta_{\rm acc}}^{\pi} \frac{d\sigma}{d\Omega} \sin\theta \, d\theta$	$I_{\rm out} = 2\pi \int_{\theta_{\rm acc}}^{\pi} \frac{d\sigma}{d\Omega} \sin\theta \, d\theta$
$A_{\rm out} = \pi \int_{\theta_{\rm acc}}^{\pi} (A_{\rm XX} + A_{\rm YY}) \frac{d\sigma}{d\Omega} \sin\theta \, d\theta$	$A_{\rm out} = 2\pi \int_{\theta_{\rm acc}}^{\pi} A_{\rm ZZ} \frac{d\sigma}{d\Omega} \sin\theta \, d\theta$
$A_{\rm all} = \pi \int_{\theta_0}^{\pi} (A_{\rm XX} + A_{\rm YY}) \frac{d\sigma}{d\Omega} \sin\theta \, d\theta$	$A_{\rm all} = 2\pi \int_{\theta_0}^{\pi} A_{\rm ZZ} \frac{d\sigma}{d\Omega} \sin\theta \, d\theta$
$K_{\rm in} = \pi \int_{\theta_0}^{\theta_{\rm acc}} (K_{\rm XX} + K_{\rm YY}) \frac{d\sigma}{d\Omega} \sin\theta \, d\theta$	$K_{\rm in} = 2\pi \int_{\theta_0}^{\theta_{\rm acc}} K_{\rm ZZ} \frac{d\sigma}{d\Omega} \sin\theta \, d\theta$
$D_{\rm in} = \pi \int_{\theta_0}^{\theta_{\rm acc}} (D_{\rm XX} + D_{\rm YY}) \frac{d\sigma}{d\Omega} \sin\theta \, d\theta$	$D_{\rm in} = 2\pi \int_{\theta_0}^{\theta_{\rm acc}} D_{\rm ZZ} \frac{d\sigma}{d\Omega} \sin\theta \, d\theta$

5.2 Solution of the system

The time dependence of the polarization of the beam is given by solving the coupled system of differential equations, leading to

$$P(t) = \frac{J(t)}{N(t)} = -\,P_{\mathrm{e}}\,\frac{A_{\mathrm{all}} - K_{\mathrm{in}}}{L_{\mathrm{in}} + L_{\mathrm{d}}\coth\left(L_{\mathrm{d}}\,n\,\nu\,t\right)}$$

where the discriminant of the quadratic equation for the eigenvalues is

$$L_{\mathrm{d}} = \sqrt{P_{\mathrm{e}}^{2}\,A_{\mathrm{out}}\left(A_{\mathrm{all}} - K_{\mathrm{in}}\right) + L_{\mathrm{in}}^{2}}$$

and $L_{\mathrm{in}} = \left(I_{\mathrm{in}} - D_{\mathrm{in}}\right)/\,2$ is the loss of polarization quantity. The approximate rate of change of polarization for sufficiently short times, and the limit of the polarization for large times are respectively:

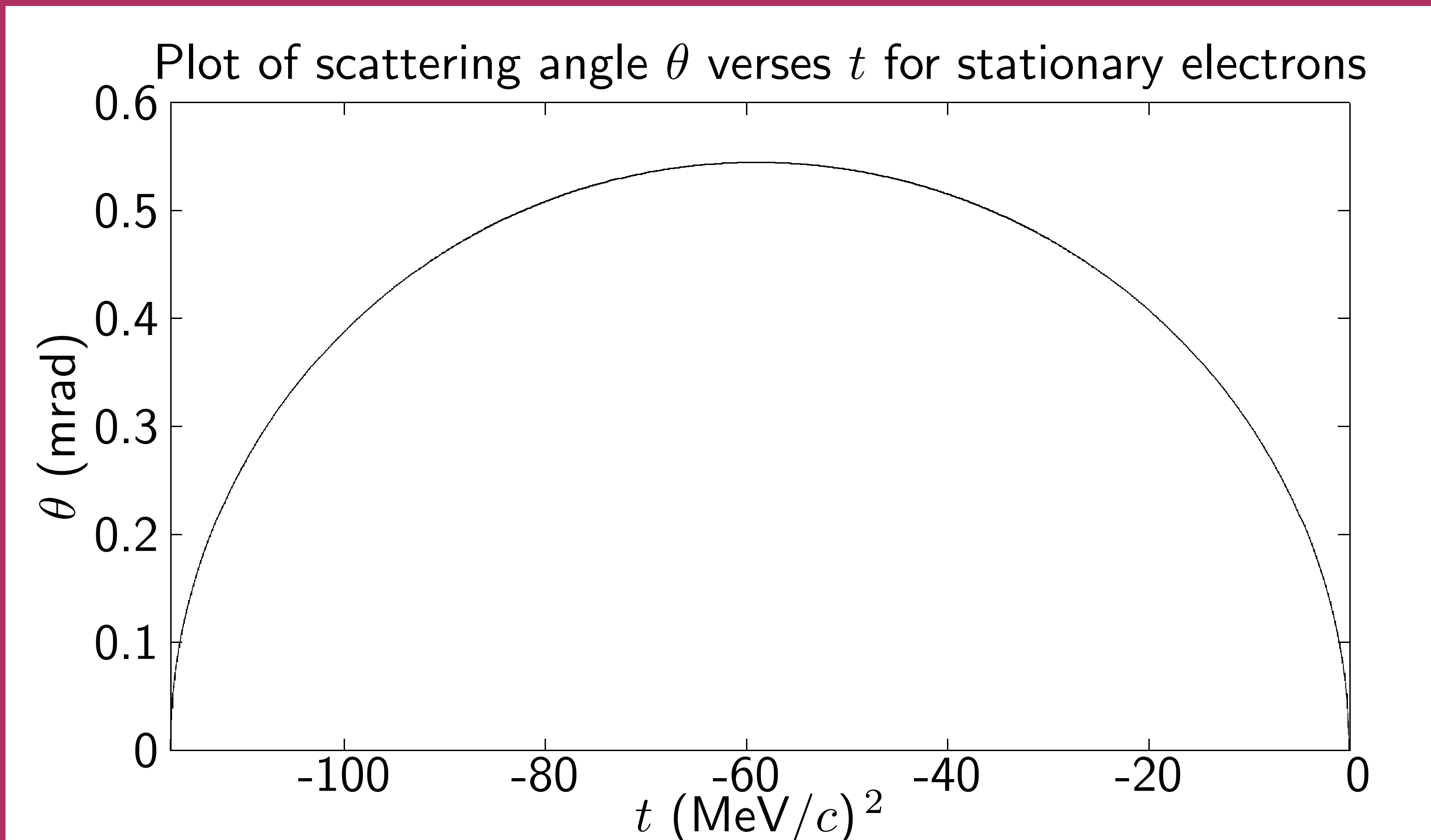
$$\frac{dP}{dt} \approx -\,n\,\nu\,P_{\mathrm{e}}\left(A_{\mathrm{all}} - K_{\mathrm{in}}\right) \qquad\qquad \lim_{t\to\infty} P(t) = -\,P_{\mathrm{e}}\,\frac{A_{\mathrm{all}} - K_{\mathrm{in}}}{L_{\mathrm{in}} + L_{\mathrm{d}}}\,.$$

5.3 Advantages of a lepton target

There are many advantages of using a polarized lepton target (or beam) over a polarized internal atomic target (hydrogen or deuterium).

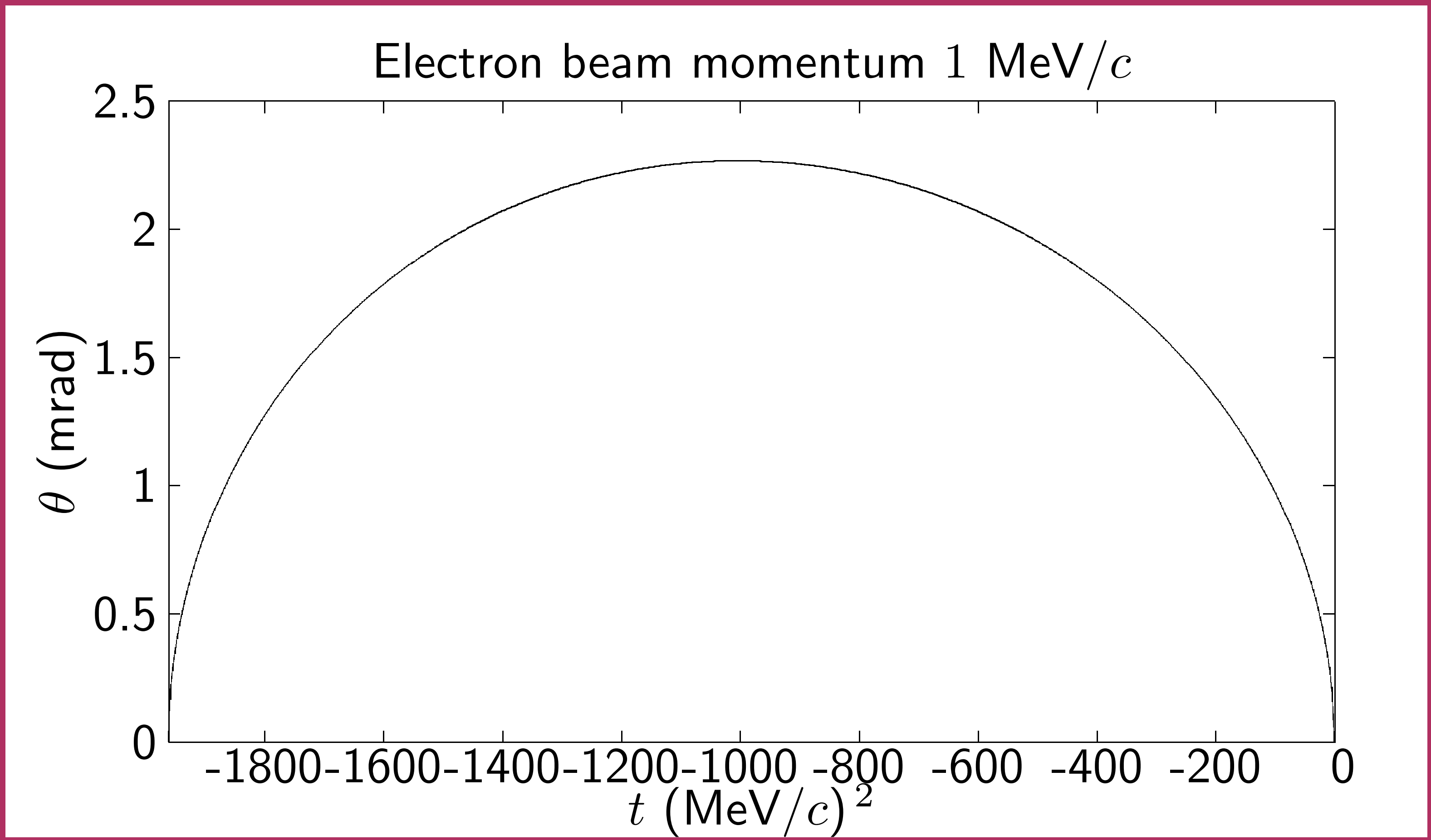
1. There is no loss of beam intensity due to annihilation of the antiprotons with protons as there is in the nuclear targets.
2. The polarization observables for antiproton - electron scattering are calculable in perturbative QED, whereas for an atomic target currently less known hadronic polarization observables contribute.
3. Residual gas does not build up in the storage ring as quickly as it would if an atomic target was used.
4. The antiproton beam could be accelerated first and then polarized, or polarized while accelerating, without worrying about decreasing hadronic spin transfer cross sections at high energy.

5.4 Scattering is within the ring for stationary electrons

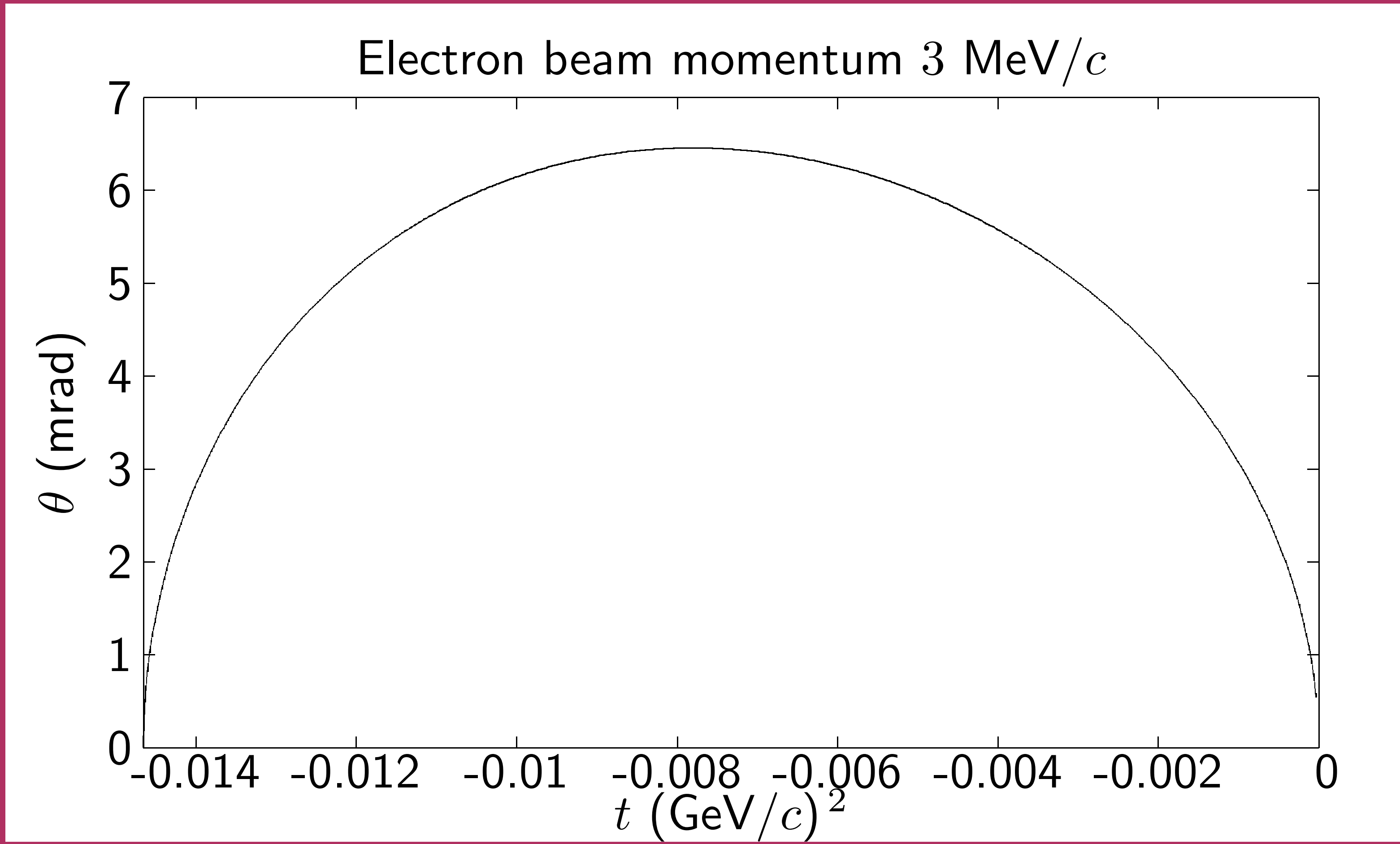


$$\theta_{\max} \;=\; m\,/\,M \;=\; 0.54 \text{ mrad} \;<\; \theta_{acc}$$

Effect of electron beam momentum (I)



Effect of electron beam momentum (II)



5.5 So what is the problem?

Although spin filtering off an opposing polarized electron beam appears to be an attractive method, due to the low electron beam areal densities currently achievable this method may not be viable. Similar conclusions were reached by Milstein and Strakhovenko (2005). Polarization buildup would take many years unless great advances in the densities of electron beams are made, which may happen in the near future considering the immense R&D effort for the ILC.

Th. Walcher *et al.* arXiv:0706.3765 (2007) propose using a co-moving polarized positron beam to transfer polarization to an antiproton beam.

Another possible suggestion would be to use a polarized muon beam/target. Antiprotons would be scattered out of the ring by stationary muons due to their much higher rest mass than electrons. Unfortunately low areal densities of a muon beam/target would also make this method unviable with current technologies.

5.6 Depolarization by electron cooling

The polarization of a beam will decrease continuously on interaction with an unpolarized electron cooler beam. The polarization half-life is:

$$\tau_{\frac{1}{2}} \;=\; \frac{\ln 2}{2\, n\, \nu\, L_{\mathrm{d}}}.$$

But the low electron areal densities in cooler beams, where typically $n \approx 10^{-18}$ fm^{-2}, causes the polarization half-life to be very large. Thus this depolarization effect is negligible, in agreement with L. W. Anderson PRD **33** 2022 (1986) and Th. Walcher *et al.* arXiv:0706.3765 (2007).

6 Scenarios of spin filtering

We have recently presented and solved systems of equations which describe polarization buildup by spin filtering in the following scenarios: (D.O'B. and N. H. Buttimore arXiv:0707.2065, Submitted to EPJA)

1. Spin filtering of a fully stored beam.
2. Spin filtering while the beam is being accumulated, $i.e.$ unpolarized particles are continuously being fed into the beam.
3. The particle input rate is equal to the rate at which particles are being lost due to scattering beyond ring acceptance angle, the beam intensity remaining constant.
4. Increasing the initial polarization of a stored beam by spin filtering.
5. The input of particles into the beam is stopped after a certain amount of time, but spin filtering continues.

Conclusions

- All Helicity Amplitudes and Spin Observables for elastic spin 1/2 - spin 1/2 scattering have been presented to first order in QED.
- In principle, an opposing beam of polarized electrons could be used to increase the polarization of an antiproton beam by spin filtering. But this method may not be viable with current technologies.
- Systems of equations have been presented and solved describing polarization buildup by spin filtering in a number of scenarios.
- We are now numerically comparing and contrasting the polarization buildup time for each of these scenarios.

Acknowledgments

I thank IRCSET for a postgraduate research scholarship.

Extra Slides

In the case of proton proton elastic scattering at a laboratory kinetic energy of 23 MeV Meyer estimates from the SAID database that the purely hadronic double spin asymmetry parameter has the value

$$A^{\mathrm{pp}}_{\mathrm{all}} = 122 \text{ mb}$$

and what is more relevant, that for collisions due to electromagnetic and hadronic interactions beyond the acceptance angle the double asymmetry parameter is

$$A^{\mathrm{pp}}_{\mathrm{out}} = 83 \text{ mb}$$

while the purely hadronic $A^{\mathrm{pp}}_{\mathrm{in}}$ is negligible.

Noting that all proton electron scattering occurs within the acceptance angle, Horowitz and Meyer evaluated the polarization transfer parameter due to proton electron elastic scattering as

$$K_{\rm in}^{\rm pe} = -70\,\rm mb$$

and the asymmetry parameter resulting from elastic proton proton collisions below the acceptance angle

$$K_{\rm in}^{\rm pp} = 52\,\rm mb$$

4.2 Helicity Amplitudes

The notation of the helicity amplitudes $\mathcal{M}(\,A', B'\,;\,A, B\,)$ is $\mathcal{M}(\,\pm, \pm\,;\,\pm, \pm\,)$ where the arguments are $+$ if the spin vector is as S_i^L above (polarized along the direction of motion) and $-$ if the spin vector is minus S_i^L above (polarized opposite to the direction of motion). After using T and P invariance there are 6 independent helicity amplitudes for the scattering of two non-identical spin $1/2$ particles.

$$\begin{array}{lclclcl}
\phi_1 & \equiv & \mathcal{M}(+,+\,;+,+) & \qquad & \phi_2 & \equiv & \mathcal{M}(+,+\,;-,-) \\
\phi_3 & \equiv & \mathcal{M}(+,-\,;+,-) & & \phi_4 & \equiv & \mathcal{M}(+,-\,;-,+) \\
\phi_5 & \equiv & \mathcal{M}(+,+\,;+,-) & & \phi_6 & \equiv & \mathcal{M}(+,+\,;-,+)
\end{array}$$

Note for $p\,p$, $\bar{p}\,p$ and $\bar{p}\,\bar{p}$ scattering $\phi_6 = -\phi_5$, so there are only 5 independent helicity amplitudes.

4.3 Helicity Amplitudes - 1st order QED results

$$\frac{\phi_1}{\alpha} = \frac{s - m^2 - M^{\,2}}{t}\left(1 + \frac{t}{4\,k^2}\right) f_1\,F_1 - f_1\,F_1 - f_2\,F_1 - f_1\,F_2 - \frac{1}{2}\,f_2\,F_2\left(1 - \frac{t}{4\,k^2}\right)$$

$$\frac{\phi_2}{\alpha} = \frac{1}{2}\left(\frac{m}{k}\,f_1 - \frac{k}{m}\,f_2\right)\left(\frac{M}{k}\,F_1 - \frac{k}{M}\,F_2\right) + \frac{s - m^2 - M^{\,2} - 2\,k^2}{4\,m\,M}\left(1 + \frac{t}{4\,k^2}\right) f_2\,F_2$$

$$\frac{\phi_3}{\alpha} = \left[\frac{s - m^2 - M^{\,2}}{t}\,f_1\,F_1 + \frac{f_2\,F_2}{2}\right]\left(1 + \frac{t}{4\,k^2}\right)$$

$$\phi_4 = -\phi_2$$

$$\frac{\phi_5}{\alpha} = \sqrt{\frac{s}{-t}\,(4\,k^2 + t)}\left[\frac{f_1\,F_1\,m}{4\,k^2}\left(1 - \frac{m^2 - M^{\,2}}{s}\right) - \frac{f_2\,F_1}{2\,m} + \frac{t\,f_2\,F_2}{16\,m\,k^2}\left(1 + \frac{m^2 - M^{\,2}}{s}\right)\right]$$

$$\frac{\phi_6}{\alpha} = \sqrt{\frac{s}{-t}\,(4\,k^2 + t)}\left[\frac{f_1\,F_1\,M}{4\,k^2}\left(\frac{M^{\,2} - m^2}{s} - 1\right) + \frac{f_1\,F_2}{2\,M} - \frac{t\,f_2\,F_2}{16\,M\,k^2}\left(1 + \frac{M^{\,2} - m^2}{s}\right)\right]$$

Define electromagnetic form factors $F_1(q^2)$ and $F_2(q^2)$, with normalization $F_1(0) = 1$ and $F_2(0) = \mu - 1$, the anomalous magnetic moment, where $q^2 = t$. We use the Sach's electric and magnetic form factors $G_M = F_1 + F_2$ and $G_E = F_1 + \dfrac{t}{4\,M^{\,2}}\,F_2$ respectively.

The differential cross section is related to the helicity amplitudes $\mathcal{M}(\Lambda', \lambda'; \Lambda, \lambda)$ by

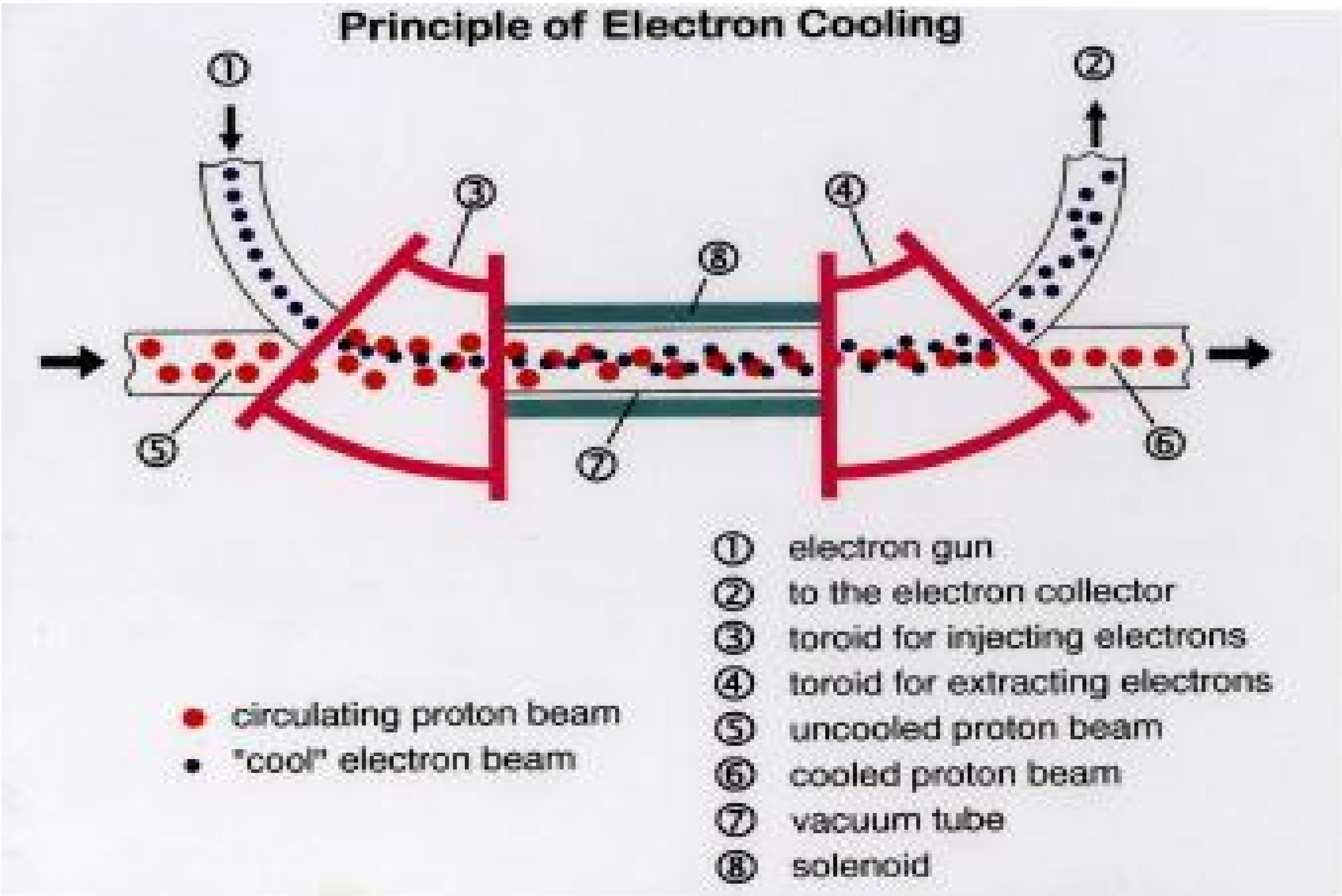
$$s\,\frac{d\sigma}{d\Omega} = \frac{1}{(8\pi)^2} \sum_{\lambda\lambda'\Lambda\Lambda'} \frac{1}{4}\,|\mathcal{M}(\Lambda', \lambda'; \Lambda, \lambda)|^2$$

where λ, Λ and λ', Λ' are the helicities of the initial and final particles respectively. The electron current is

$$j^\mu = e\,\bar{u}(p_4, \lambda')\,\gamma^{\,\mu}\,u(p_2, \lambda)\,,$$

and the antiproton current, after Gordon decomposition is

$$J^\mu = e_{\bar{p}}\;\bar{u}(p_3, \Lambda')\left(G_M\,\gamma^{\,\mu} - F_2\,\frac{p_2^{\,\mu} + p_4^{\,\mu}}{2M}\right)u(p_1, \Lambda)\,.$$



Loss of antiproton beam polarization is negligable in an electron cooler.