Formulas:

- The real Fourier series expansion of a function \( f(t) \) of fundamental period \( L \) can be written as
  \[
f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \left( \frac{2\pi nt}{L} \right) + \sum_{n=1}^{\infty} b_n \sin \left( \frac{2\pi nt}{L} \right),
\]
  where the coefficients are given by the Euler formulas:
  \[
a_0 = \frac{2}{L} \int_{-L/2}^{L/2} f(t) \, dt
  
  a_n = \frac{2}{L} \int_{-L/2}^{L/2} f(t) \cos \left( \frac{2\pi nt}{L} \right) \, dt
  
  b_n = \frac{2}{L} \int_{-L/2}^{L/2} f(t) \sin \left( \frac{2\pi nt}{L} \right) \, dt
  
- Parseval’s Theorem: For a function of period \( L \) whose real Fourier series expansion is written in the form above, the following equation is true:
  \[
  \frac{1}{L} \int_{t_0}^{t_0+L} f(t)^2 \, dt = \left( \frac{a_0}{2} \right)^2 + \frac{1}{2} \sum_{n=1}^{\infty} (a_n^2 + b_n^2).
  
- Fourier Transform:
  \[
  \tilde{f}(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) e^{-i\omega t} \, dt.
  
Questions:

1. In Tutorial Sheet 2, we computed the Fourier series of the following function:
   \[
   f(t) = \begin{cases} 
   t^2 & \text{for } |t| < 1, \\
   f(t+2) & \text{otherwise}.
   \end{cases}
   
   The following Fourier coefficients were obtained:
   \[
   a_0 = \frac{2}{3}, \quad a_n = \frac{4(-1)^n}{\pi^2 n^2}, \quad b_n = 0.
   
   Use Parseval’s Theorem to evaluate the sum of the infinite series
   \[
   \sum_{n=1}^{\infty} \frac{1}{n^4}.
   
2. Using the definition given above, compute the Fourier transform of \( f(t) = te^{-2|t|} \).